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INVESTIGACIÓN

Analysis of the Instantaneous Center of Rotation of a Six-Bar Mechanism Using Nodal Coordinates

Análisis del Centro Instantáneo de Rotación de un Mecanismo de Seis Barras usando Coordenadas Nodales

Neider Nadid Romero N.[1], Gonzalo Moreno Contreras[2], Rafael Villalba González[3], Daniel Martins[4]

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Abstract

Objective: Knowledge of the instantaneous center associated with two bodies in a mechanism is important for both analy-sis and synthesis, because it can simplify velocity and acceleration analysis. Moreover, accurate determination of the instantaneous center position is a key requirement for synthesis in many practical applications. The purpose of this work is to propose an analytical method for calculating the instantaneous center of rotation of a floating link in a six-bar mechanism with respect to the fixed link.

Methodology: The proposed procedure consists of the following steps. First, the variables representing the mechanism dimensions are defined. Unlike traditional approaches, this work uses initial nodal coordinates. The constraint equations are then formulated using both the current and initial nodal coordinates. Next, the Levenberg-Marquardt method is applied to solve the constraint equations robustly. Finally, the kinematic constraint equations for the instantaneous centers are established using the Aronhold-Kennedy theorem and solved to determine the coordinates of the desired instantaneous center.

1 Mechanical Engineer (University of Pamplona, Colombia); M.Sc. and Ph.D. in Mechanical Engineering – Mechanical Sys-tems Design (Federal University of Santa Catarina, Brazil). Currently full-time professor at the University of Pamplona, working in multibody system dynamics and optimal design of mechanisms. Email: neider.romero@unipamplona.co

2 Mechanical Engineer (Francisco de Paula Santander University, Colombia); M.Sc. in Mechanical Engineering (University of the Andes, Colombia); Ph.D. in Mechanical Engineering (Federal University of Santa Catarina, Brazil). Currently pro-fessor in the Mechanical Engineering Department at the University of Pamplona. Email: gmoren@unipamplona.edu.co

3 Mechanical Engineer (University of Pamplona, Colombia); currently pursuing a specialization in Integrated Management Systems (HSEQ). Currently professor in the Mechanical Engineering program at the University of Pamplona, working on design and optimization of steering mechanisms. Email: rafael.villalba@unipamplona.edu.co

4 B.Sc., M.Sc., and Ph.D. in Mechanical Engineering (Federal University of Santa Catarina, Brazil); Postdoctoral fellow (King's College London). Currently full professor at the Federal University of Santa Catarina (UFSC), specializing in me-chanism design, robotics, and intellectual property. Email: daniel.martins@ufsc.br

Results: Using nodal coordinates and the Aronhold-Kennedy theorem, a procedure was developed to determine the ins-tantaneous center of rotation of a link with respect to the fixed link of a six-bar mechanism. The proposed method is sufficiently robust for use in optimal synthesis processes, which will be addressed in future work.
Conclusions: A method was developed to determine the instantaneous center in a six-bar mechanism using nodal coordinates. The method was validated by implementing the proposed procedure in a mechanism with arbitrary dimensions and comparing the results with those obtained using GIM (Geometric Interactive Method).
Keywords: Instantaneous center, nodal coordinates, six-bar mechanism, Aronhold-Kennedy theorem.

Resumen

Objetivo: El conocimiento del centro instantáneo asociado a dos cuerpos en un mecanismo es importante tanto para el análisis como para la síntesis, dado que puede simplificar el análisis de velocidades y aceleraciones. Asimismo, la determinación precisa de la posición del centro instantáneo es un requisito clave para la síntesis en muchas aplicaciones prácticas. El propósito de este trabajo es proponer un método analítico para calcular el centro instantáneo de rotación de un eslabón flotante en un mecanismo de seis barras con respecto al eslabón fijo.
Metodología: El procedimiento propuesto consta de los siguientes pasos. En primer lugar, se definen las variables que representan las dimensiones del mecanismo. A diferencia de los enfoques tradicionales, este trabajo emplea coordenadas nodales iniciales. A continuación, se formulan las ecuaciones de restricción utilizando tanto las coordenadas nodales actuales como las iniciales. Posteriormente, se aplica el método de Levenberg-Marquardt para resolver de manera robusta las ecuaciones de restricción. Finalmente, se establecen las ecuaciones de restricción cinemática para los centros instantáneos mediante el teorema de Aronhold-Kennedy y se resuelven para determinar las coordenadas del centro instantáneo deseado.
Resultados: Utilizando coordenadas nodales y el teorema de Aronhold-Kennedy, se desarrolló un procedimiento para determinar el centro instantáneo de rotación de un eslabón con respecto al eslabón fijo de un mecanismo de seis barras. El método propuesto es suficientemente robusto para ser empleado en procesos de síntesis óptima, los cuales serán abordados en trabajos futuros.
Conclusiones: Se desarrolló un método para determinar el centro instantáneo en un mecanismo de seis barras utilizando coordenadas nodales. El método fue validado mediante la implementación del procedimiento propuesto en un mecanismo con dimensiones arbitrarias y la comparación de los resultados con los obtenidos mediante GIM (Método Interactivo Geométrico).
Palabras clave: Centro instantáneo, coordenadas nodales, mecanismo de seis barras, teorema de Aronhold-Kennedy.

Introduction

The computation of instantaneous centers of rotation (ICRs) is crucial in several aspects of pla-nar mechanism analysis. ICRs are particularly important for velocity and singularity analysis (1, 2), because they support the prediction of mechanism behavior under different operating conditions. Accurate ICR identification is also essential for configuration synthesis, allowing designers to create mechanisms that satisfy specific motion requirements. In addition, a clear understanding of ICRs is necessary for dynamic modeling, as it provides a basis for accurate simulations and predictions of mechanism behavior under dynamic loads (3). The importance of this analysis is reflected in the many publications devoted to refining and extending these techniques, particularly for complex me-chanisms for which traditional methods are insufficient.

Kennedy's theorem is a fundamental tool in planar mechanism kinematics, providing a graphical method for locating instantaneous centers of rotation (ICRs) in four-bar linkages (1, 2, 4). The theorem is based on the principle that the ICRs of three links in relative motion are collinear. By identifying the intersection of lines connecting known ICRs, the remaining ICRs can be determined systemati-cally. This geometric approach is particularly useful for simpler mechanisms, as it offers an intuitive visualization of instantaneous kinematic behavior. The simplicity and efficiency of the theorem have made it a standard topic in introductory kinematics courses and textbooks.

However, Kennedy's theorem has significant limitations when applied to more complex planar mechanisms. Its applicability is strongly restricted for mechanisms beyond the simple four-bar configuration. For multidegree-of-freedom (multi-DOF) linkages and mechanisms classified as complex or indeterminate, Kennedy's theorem often fails to identify all ICRs (4–6). This limitation stems from the theorem's reliance on readily identifiable four-bar loops within the mechanism structure. In me-chanisms with insufficient four-bar loops or intricate interconnections, the graphical approach of Kennedy's theorem becomes inadequate. The distinction between primary and secondary instant centers further complicates its application. Primary ICRs can often be identified directly by inspec-ting the mechanism geometry, whereas secondary ICRs require more sophisticated techniques (2, 6). The difficulty of locating secondary ICRs using purely graphical methods underscores the need for alternative, more robust approaches. Therefore, the limitations of Kennedy's theorem highlight the need for advanced techniques capable of handling the complexities of multi-DOF and indeterminate linkages.

Screw theory provides a powerful algebraic framework for analyzing the kinematics of planar and spatial mechanisms. This approach has been extended to planar mechanisms, offering an alterna-tive method for determining ICRs, particularly in indeterminate linkages. Valderrama Rodríguez (7) demonstrates the applicability of screw theory to both planar and spherical indeterminate mechanisms. This approach simplifies the equations involved compared with earlier methods, making it more computationally efficient.

Another approach for determining instantaneous centers is based on constraint equations. These equations can be greatly simplified through the use of natural coordinates (8–10) and can then be solved using numerical or analytical methods. In the literature, this methodology is documented only in the work of Sancibrian R. et al. (11), where it is applied to the optimal design of motorcycle suspensions, knee prosthesis mechanisms, and mechanical advantage mechanisms. It is important to note that, although that study uses constraint equations, it does not use natural coordinates.

This paper presents a robust methodology for analyzing the instantaneous centers of a six-bar mechanism using nodal coordinates. The main idea is to formulate Kennedy's theorem in terms of kinematic constraints, which are solved numerically using the Levenberg-Marquardt method. This formulation makes it possible to obtain solutions even when the mechanism reaches singular posi-tions. The results are compared with those obtained from a kinematic analysis program, demonstra-ting the effectiveness of the method. Although the technique is not directly applicable to indetermi-nate mechanisms, the authors believe that combining this methodology with screw theory may make it possible to solve any planar mechanism; this topic will be studied in future work.

Instantaneous center of rotation

An instantaneous center of rotation is defined as a point common to two bodies in plane motion that has the same instantaneous velocity in both bodies (14). Aronhold and Kennedy independently proved that the instantaneous centers of three bodies are aligned as stated in Theorem 1. In mathema-tical terms, the Aronhold-Kennedy theorem can be expressed by Equation 1, where $\boldsymbol{\omega}_{ij}$ is the angular velocity of body j with respect to body i.

Theorem 1 (Aronhold-Kennedy theorem) *Any three bodies in plane motion will have exactly three instantaneous centers, and they will lie on the same straight line.*

$$\boldsymbol{\omega}_{ik} \times \overrightarrow{P_{ik}P_{ij}} = \boldsymbol{\omega}_{jk} \times \overrightarrow{P_{jk}P_{ij}} \qquad (1)$$

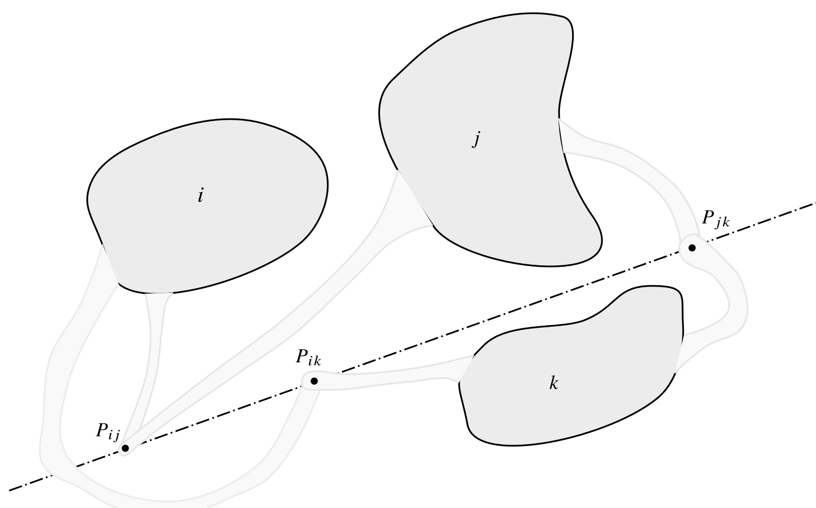


Figure 1. Graphical representation of the Aronhold-Kennedy theorem.

Source: Authors.

In practical terms, the ICR represents the point around which a body appears to rotate at a given instant, even when the body is undergoing a complex motion combining translation and rotation. This fundamental kinematic property makes it possible to significantly simplify the analysis of velocities and accelerations in complex mechanical systems, converting general motions into instanta-neous pure rotations.

Kinematic modeling

This section presents the formulation used to determine instantaneous centers using nodal coor-dinates. For didactic purposes, the step-by-step procedure is first shown for a four-bar mechanism and is then extended to a six-bar mechanism, which is the objective of this work.

Four-bar mechanism

In this section, the constraint equations that define the kinematics of the mechanism will be deve-loped. For this purpose, the following dimensions are defined: x^0, y^0, x^0, y^0, x_A, y_A, x_B, y_B, where the first four design variables correspond to the initial coordinates of the pairs P_1 and P_2, respectively, and the remaining four variables are the coordinates of the fixed pivots A and B, see Figure 2. The dimensions can be represented in a compact form using a vector,

$$\mathbf{z} = [x_1^0\, y_1^0\, x_2^0\, y_2^0\, x_A\, y_A\, x_B\, y_B]^T \qquad (2)$$

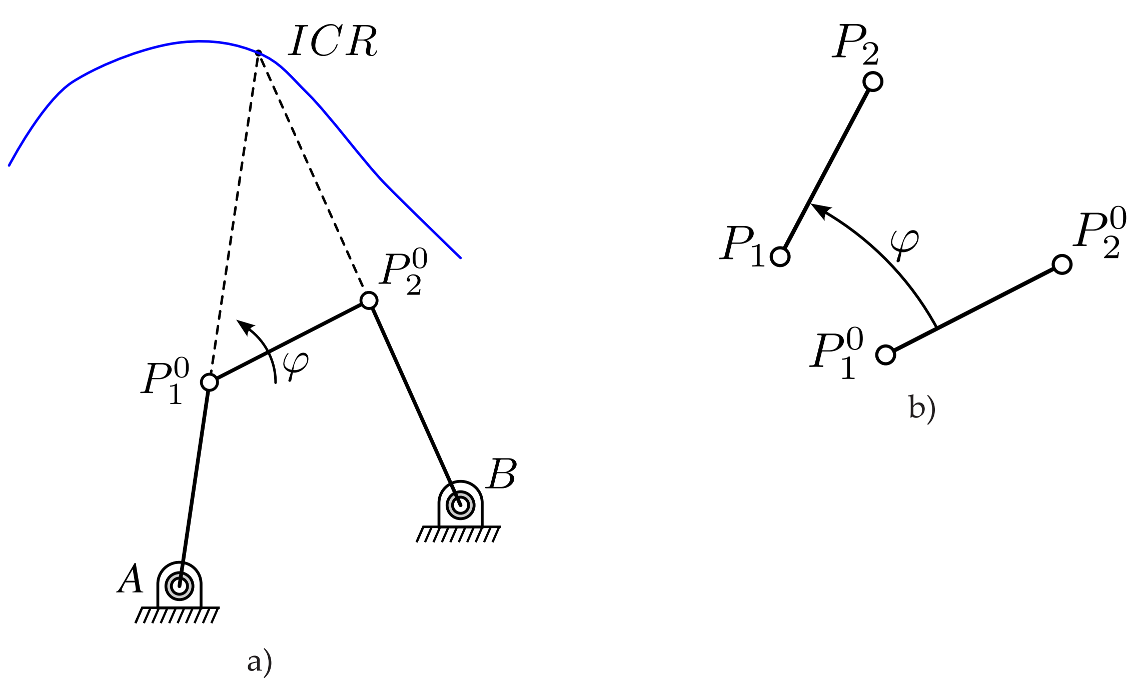


Figure 2. Kinematic representation of the four-bar mechanism.

Source: Authors.

where **z** is the vector of dimensions. To avoid ambiguity in the subsequent formulation, it is emphasized that x^0, y^0, x^0, and y^0 denote the initial coordinates of points P_1 and P_2, whereas x_1, y_1, x_2, and y_2 denote their current coordinates after the coupler rotates by the input angle (denoted here by φ).

Links AP_1 and BP_2 are rigid, so their respective distances remain constant. This can be expressed mathematically as:

$$(x_1 - x_A)^2 + (y_1 - y_A)^2 - [(x^0{}_1 - x_A/2 + (y^0{}_1 - y_A)^2] = 0 \qquad (3)$$

$$(x_2 - x_B)^2 + (y_2 - y_B)^2 - [(x^0{}_2 - x_B/2 + (y^0{}_2 - y_B)^2] = 0 \qquad (4)$$

where Equations 3 and 4 are simply circles with centers at A and B with radii d_{A1} and d_{B2}, respectively. Now note that points P_1 and P_2 are not independent since the coupling link is also rigid, and these two points are dependent on the input angle φ. Therefore, a rotation of an angle φ in the seg-ment P^0P^0 preserves the condition of rigidity, see Figure 2(b), thus, the rotation of the segment can be expressed as:

$$\begin{bmatrix} (x_2 - x_1) \\ (y_2 - y_1) \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} (x_2^0 - x_1^0 \\ (y_2^0 - y_1^0 \end{bmatrix} \qquad (5)$$

which can be written as follows,

$$(x_2 - x_1) - (x_2^0 - x_1^0)\cos\varphi + (y_2^0 - y_1^0)\sin\varphi = 0 \qquad (6)$$

$$(y_2 - y_1) - (x_2^0 - x_1^0)\sin\varphi - (y_2^0 - y_1^0)\cos\varphi = 0 \qquad (7)$$

Equations 3, 4, 6 and 7 represent the kinematics of the four-bar mechanism with the coupler as the input link. These constraint equations can be written compactly as follows,

$$\Phi(q, z) = \begin{bmatrix} (x_1 - x_A)^2 + (y_1 - y_A)^2 - \left[(x_1^0 - x_A)^2 + (y_1^0 - y_A)^2\right] \\ (x_2 - x_B)^2 + (y_2 - y_B)^2 - \left[(x_2^0 - x_B)^2 + (y_2^0 - y_B)^2\right] \\ (x_2 - x_1) - (x_2^0 - x_1^0)\cos\phi + (y_2^0 - y_1^0)\sin\phi \\ (y_2 - y_1) - (x_2^0 - x_1^0)\sin\phi - (y_2^0 - y_1^0)\cos\phi \end{bmatrix} = 0 \qquad (8)$$

where **q** is the vector of natural coordinates that corresponds to the coordinates of points P_1 and P_2; therefore,

$$q = [x_1 \quad y_1 \quad x_2 \quad y_2]^T \qquad (9)$$

Equation 8 can be solved efficiently using the Newton-Raphson method, which has quadratic convergence provided that the initial estimate is close to the solution. This approach is adequate for simulations, but it is not sufficiently robust for mechanism optimization, because the optimization process may generate mechanism dimensions that are not physically feasible, i.e., cases in which assembly is impossible. To address this problem, the position problem can be reformulated as the minimization of the sum of squares:

$$\underset{\mathrm{q}}{\text{minimize}} \quad \frac{1}{2}\Phi(\mathrm{q},\mathrm{z})^T\Phi(\mathrm{q},\mathrm{z}) \qquad (10)$$

which provides an error measure when mechanism assembly is not physically possible. The re-si-dual error can then be used to penalize the objective function, making gradient-based optimization methods feasible (12). The optimization problem formulated in Equation 10 can be solved using the Levenberg-Marquardt method, as shown in Algorithm 1.

Algorithm 1 Levenberg-Marquardt method

Require: $\boldsymbol{\Phi}_{\mathbf{q}} = \partial\boldsymbol{\Phi}_{\mathbf{q}}/\partial\mathbf{q}$, λ, $\mathbf{q}_0$, $\boldsymbol{\Phi}$

Ensure: q

for j=1 to 20 **do**
  $\Delta f \leftarrow \boldsymbol{\Phi}_{\mathrm{q}}^T \cdot \boldsymbol{\Phi}$;
  $\mathbf{H} \leftarrow \frac{\partial \nabla f}{\partial q}$
  if $\|\nabla f\| < \epsilon$ **then**
    break
  end if
  $(\mathbf{H}+\lambda\mathbf{I})\mathbf{s} \leftarrow -\nabla f$
  $f_p \leftarrow f(\mathbf{q}+\mathbf{s})$
  if $f_p < f$ **then**
    $\lambda \leftarrow \lambda/10$
    $\mathbf{q} \leftarrow \mathbf{q}+\mathbf{s}$
  else
    $\lambda \leftarrow \lambda \cdot 10$
  end if
end for

In Algorithm 1, **H** is the Hessian matrix, **I** is the identity matrix, λ is a scalar value, and Δf is the gradient.

Once the position kinematics of the four-bar mechanism has been solved, the instantaneous center of rotation (ICR), illustrated in Figure 2, must be determined:

$$\mathbf{r}AP^{ic} \times \mathbf{r}AP_1 = 0 \qquad (11)$$

$$\mathbf{r}BP^{ic} \times \mathbf{r}BP_2 = 0 \qquad (12)$$

where P^{ic} is the position of the instantaneous center of rotation between the coupler and the fixed link. The above two constraint equations can be written in expanded form as,

$$(x^{ic} - x_A)(y_1 - y_A) - (y^{ic} - y_A)(x_1 - x_A) = 0 \qquad (13)$$

$$(x^{ic} - x_B)(y_2 - y_B) - (y^{ic} - y_B)(x_2 - x_B) = 0 \qquad (14)$$

These two constraint equations can be rewritten as follows,

$$(y_1 - y_A)x^{ic} - (x_1 - x_A)y^{ic} = (y_1 - y_A)x_A - (x_1 - x_A)y_A \qquad (15)$$

$$(y_2 - y_B)x^{ic} - (x_2 - x_B)y^{ic} = (y_2 - y_B)x_B - (x_2 - x_B)y_B \qquad (16)$$

Now Equations 15 and 16 can be written in matrix form as follows,

$$\begin{bmatrix} (y_1 - y_A) & -(x_1 - x_A) \\ (y_2 - y_B) & -(x_2 - x_B) \end{bmatrix} \begin{bmatrix} x^{ic} \\ y^{ic} \end{bmatrix} = \begin{bmatrix} (y_1 - y_A)x_A - (x_1 - x_A)y_A \\ (y_2 - y_B)x_B - (x_2 - x_B)y_B \end{bmatrix} \qquad (17)$$

In order to generalize the procedures developed for the four-bar mechanism to the analysis of more complex mechanisms, Equation 17 can be rewritten as follows,

$$\mathbf{A}^{ic}(\mathbf{q})\mathbf{P}^{ic} = \mathbf{B}^{ic}(\mathbf{q}) \qquad (18)$$

where $\mathbf{A}^{ic}$ is the coefficient matrix and $\mathbf{B}^{ic}$ is the resultant vector. Equation 18 is referred to as the instantaneous-center equation. In the following section, where a six-bar mechanism is analyzed, it is shown that the instantaneous-center equation is generally nonlinear.

Six-bar mechanism

In this section, the method proposed for the four-bar mechanism is generalized to the kinematic study of six-bar mechanisms, as shown in Figure 3. This mechanism is defined by the following dimensions or design variables:

$$Z = [x_1^0\ y_1^0\ x_2^0\ y_2^0\ x_3^0\ y_3^0\ x_4^0\ y_4^0\ x_5^0\ x_5^0\ x_A\ y_A\ x_B\ y_B]^T \qquad (19)$$

The constraint equations that describe the positional kinematics of the mechanism are now defined. Distance equations can be established for the segments AP_1, P_1P_2, BP_2, P_1P_3, P_2P_3, P_2P_5, BP_5,

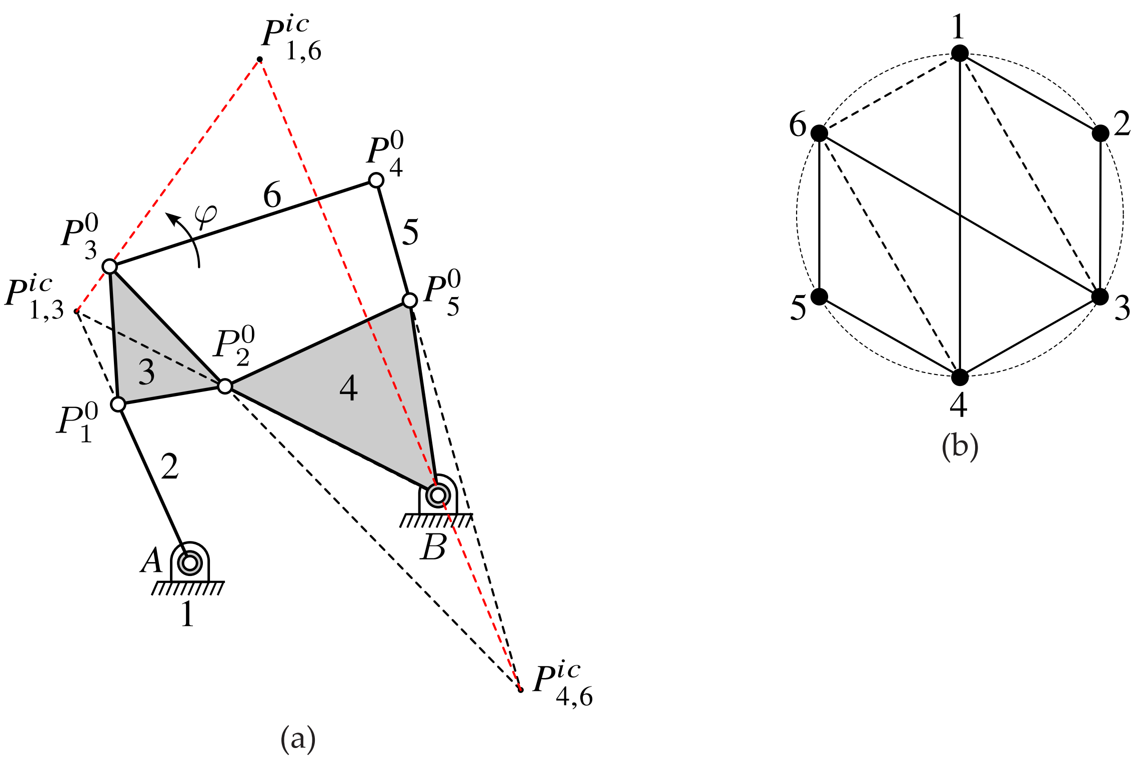


Figure 3. Kinematic diagram and location of the ICRs of the six-bar mechanism using Kennedy's theorem.

Source: Authors.

and P_4P_5. Two additional constraint equations are then defined by applying a rotation φ to the segment P_3P_4. These constraints are grouped in the constraint vector shown in Equation 21, where the vector of natural coordinates is:

$$q = [x_1\ y_1\ x_2\ y_2\ x_3\ y_3\ x_4\ y_4\ x_5\ y_5]^T \qquad (20)$$

$$\Phi(q, z) = \begin{bmatrix} (x_1 - x_A)^2 + (y_1 - y_A)^2 - \left[(x_1^0 - x_A)^2 + (y_1^0 - y_A)^2 \right] \\ (x_2 - x_B)^2 + (y_2 - y_B)^2 - \left[(x_2^0 - x_B)^2 + (y_2^0 - y_B)^2 \right] \\ (x_2 - x_1)^2 + (y_2 - y_1)^2 - \left[(x_2^0 - x_1^0)^2 + (y_2^0 - y_1^0)^2 \right] \\ (x_3 - x_1)^2 + (y_3 - y_1)^2 - \left[(x_3^0 - x_1^0)^2 + (y_3^0 - y_1^0)^2 \right] \\ (x_3 - x_2)^2 + (y_3 - y_2)^2 - \left[(x_3^0 - x_2^0)^2 + (y_3^0 - y_2^0)^2 \right] \\ (x_5 - x_2)^2 + (y_5 - y_2)^2 - \left[(x_5^0 - x_2^0)^2 + (y_5^0 - y_2^0)^2 \right] \\ (x_5 - x_B)^2 + (y_5 - y_B)^2 - \left[(x_5^0 - x_B)^2 + (y_5^0 - y_B)^2 \right] \\ (x_5 - x_4)^2 + (y_5 - y_4)^2 - \left[(x_5^0 - x_4^0)^2 + (y_5^0 - y_4^0)^2 \right] \\ (x_4 - x_3 - (x_4^0 - x_3^0) \cos\phi + (y_4^0 - y_3^0) \sin\phi \\ (y_4 - y_3) - (x_4^0 - x_3^0) \sin\phi - (y_4^0 - y_3^0) \cos\phi \end{bmatrix} = 0 \qquad (21)$$

For the mechanism under analysis, the centers of rotation can be determined graphically using Kennedy's theorem (14), which states that the instantaneous centers of three bodies are aligned. For more complex mechanisms, velocity analysis can be performed using natural coordinates (9) or screw theory, which enables systematic determination of all instantaneous centers without calculating derivatives (13).

The relationship between the instantaneous centers can be written mathematically by means of the constraint vector:

$$\mathbf{\Phi}^{ic}(\mathbf{q}, \mathbf{q}^{ic}) = \mathbf{0} \qquad (22)$$

where $\mathbf{\Phi}^{ic}$ is the vector of instantaneous-center constraints and $\mathbf{q}^{ic}$ is the vector of instantaneous centers. For the six-bar mechanism shown in Figure 3, Equation 22 can be written as:

$$\Phi^{ic}\left(q, q^{ic}\right) = \begin{bmatrix} \mathbf{r}_{AP_{1,3}^{ic}} \times \mathbf{r}_{AP_1} \\ \mathbf{r}_{BP_{1,3}^{ic}} \times \mathbf{r}_{BP_2} \\ \mathbf{r}_{P_4P_{4,6}^{ic}} \times \mathbf{r}_{P_4P_5} \\ \mathbf{r}_{P_3P_{4,6}^{ic}} \times \mathbf{r}_{P_3P_2} \\ \mathbf{r}_{BP_{4,6}^{ic}} \times \mathbf{r}_{BP_{1,6}^{ic}} \\ \mathbf{r}_{P_3P_{1,3}^{ic}} \times \mathbf{r}_{P_3P_{1,6}^{ic}} \end{bmatrix} = 0 \qquad (23)$$

In this case, the equation was found to be nonlinear with respect to the coordinates of the ins-tantaneous centers. However, the system can be decoupled into two linear systems: one with four equations and four unknowns, Equation 24, and another with two equations and two unknowns, Equation 25. Whether the general system can always be expressed as a set of linear systems remains unknown; addressing this question would require the analysis of additional systems.

$$\begin{bmatrix} (y_1 - y_A) & -(x_1 - x_A) & 0 & 0 \\ (y_2 - y_B) & -(x_2 - x_B) & 0 & 0 \\ 0 & 0 & (y_5 - y_4) & -(x_5 - x_4) \\ 0 & 0 & (y_2 - y_3) & -(x_2 - x_3) \end{bmatrix} \begin{bmatrix} x_{P_{13}} \\ y_{P_{13}} \\ x_{P_{46}} \\ y_{P_{46}} \end{bmatrix} = \begin{bmatrix} x_A (y_1 - y_A) + y_A (x_1 - x_A) \\ x_B (y_2 - y_B) + y_B (x_2 - x_B) \\ x_4 (y_5 - y_4) + y_4 (x_5 - x_4) \\ x_3 (y_2 - y_3) + y_3 (x_2 - x_3) \end{bmatrix} \tag{24}$$

$$\begin{bmatrix} -(y_{P_{46}} - y_B) & (x_{P_{46}} - x_B) \\ -(y_{P_{13}} - y_3) & (x_{P_{13}} - x_3) \end{bmatrix} \begin{bmatrix} x_{P_{16}} \\ y_{P_{16}} \end{bmatrix} = \begin{bmatrix} y_B (x_{P_{46}} - x_B) - x_B (y_{P_{46}} - y_B) \\ y_3 (x_{P_{13}} - x_3) - x_3 (y_{P_{13}} - y_3) \end{bmatrix} \tag{25}$$

Results

This section demonstrates the implementation of the proposed procedure for arbitrary dimen-sions of both a four-bar mechanism and a six-bar mechanism. In practical problems, the instantaneous center of rotation of a particular link with respect to the fixed link is often of interest. Therefore, only instantaneous centers of practical interest are analyzed here. The proposed model was imple-mented in MATLAB, and its results were verified using GIM (Geometrical Interactive Method). Figures 4 and 5 provide a graphical comparison between the results obtained with the proposed model and those obtained with GIM for the four-bar and six-bar mechanisms, respectively. For both mecha-nisms, the maximum absolute error and RMSE were zero within the numerical precision adopted in both programs. For reproducibility, the source codes and simulation files were deposited in Zenodo for the four-bar mechanism at (15) and for the six-bar mechanism at (16).

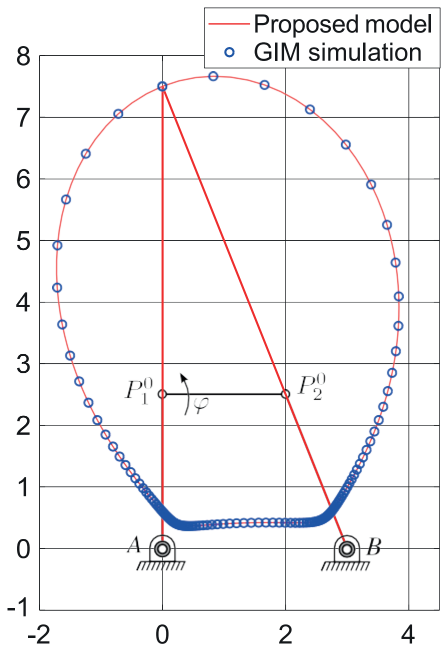
Table 1 shows the nodal dimensions of the four-bar mechanism, where all dimensions are expres-sed in units of length. Because this is an illustrative example, the units are not specified. Figure 4(a) shows, in red, the trajectory of the instantaneous center of the coupler with respect to the fixed link. Figure 4(b) illustrates a possible application of this instantaneous center calculation. This application is relevant because the trajectory of the instantaneous center of rotation is an important kinematic indicator in the analysis and design of knee prosthesis mechanisms, where the relative motion between the thigh and shank must be accurately reproduced.

Table 1. *Nodal dimensions of the four-bar mechanism.*

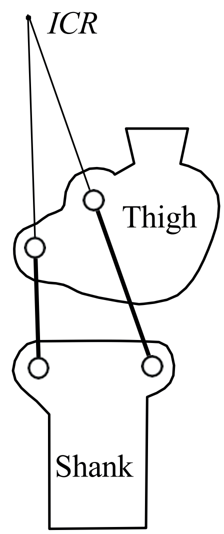
Dimensions	x_1^0	y_1^0	x_2^0	y_2^0	x_A	y_A	x_B	y_B
Values	0	2.5	2	2.5	0	0	3	0

Source: Authors.

Table 2 shows the nodal dimensions of the six-bar mechanism, where all dimensions are expressed in units of length. Figure 5(a) shows, in red, the trajectory of the instantaneous center of link 3 relative to the fixed link; in green, the instantaneous center of link 6 relative to link 4; and in blue, the instantaneous center of link 6 relative to fixed link 1. A possible application of this instantaneous center calculation is illustrated in Figure 5(b).



(a)



(b)

Figure 4. Comparison between the results obtained with the proposed MATLAB implementation and GIM (Geometrical Interactive Method) for the four-bar mechanism. (a) Trajectory of the instantaneous center of rotation of the coupler relative to the fixed link. (b) Example of application to the kinematic analysis of a knee prosthesis.

Source: Authors.

Conclusions

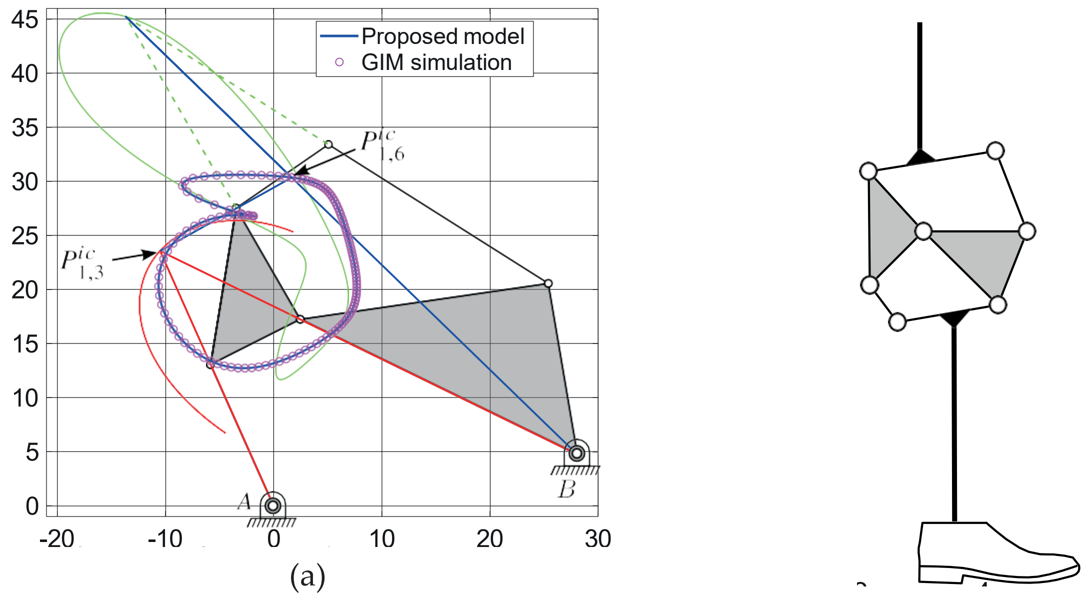
This study developed and validated an analytical method for determining the instantaneous center of rotation (ICR) in planar mechanisms, specifically in a six-bar linkage, using nodal coordinates and the Aronhold-Kennedy theorem. The proposed methodology is based on formulating kinematic constraint equations as a nonlinear system, which is solved robustly using the Levenberg-Marquardt method. The results show that this approach enables precise identification of ICRs of practical interest and can be applied to arbitrary geometries, thereby avoiding reliance on traditional graphical methods that often fail in complex or indeterminate mechanisms. In addition, it was shown that, under certain conditions, the nonlinear system can be rewritten as a set of linear subsystems, opening the door to more efficient computational implementations.

Table 2. *Nodal dimensions of the six-bar mechanism.*

Dimensions	Values	Dimensions	Values
x_1^0	-5.820789	y_1^0	13.05212
x_2^0	2.471042	y_2^0	17.23752
x_3^0	-3.451695	y_3^0	27.5036
x_4^0	5.071548	y_4^0	33.37934
x_5^0	25.37229	y_5^0	20.55425
x_A	0	y_A	0
x_B	28.05727	y_B	4.760285

Source: Authors.

This approach has strong potential for incorporation into mechanism synthesis and optimization, and it provides a solid foundation for future research in multibody system dynamics, particularly when combined with geometric frameworks such as screw theory.



(a) (b)

Figure 5. Comparison between the results obtained with the proposed MATLAB implementation and GIM (Geometrical Interactive Method) for the six-bar mechanism. (a) Trajectories of the instantaneous centers of rotation: link 3 relative to the fixed link, link 6 relative to link 4, and link 6 relative to the fixed link. (b) Example of application of the computed instantaneous centers.

Source: Authors.

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References

[1] S. Zarkandi, "Application of mechanical advantage and instant centers on singularity analysis of single-DOF planar mechanisms," *J. Mech. Eng.*, vol. 41, no. 1, pp. 50–57, 2010. https://doi.org/10.3329/jme.v41i1.5362 ↑ 2, 3

[2] P. A. Simionescu, I. Talpasanu, and R. di Gregorio, "Instant-center based force transmissivity analysis in planar mechanisms," *J. Mech. Des.*, vol. 132, no. 6, pp. 061003–1–061003–9, 2010. https://doi.org/10.1115/1.4001094 ↑ 2, 3

[3] L. Nie, H. Ding, K.-L. Ting, and A. Kecskeméthy, "Instant center identification of single-loop multi-DOF planar linkage using virtual link," *Appl. Sci.*, vol. 11, no. 10, p. 4463, 2021. https://doi.org/10.3390/app11104463 ↑ 3

[4] J. I. Valderrama-Rodríguez, J. M. Rico, J. J. Cervantes-Sánchez, and R. García-García, "A screw theory approach to computing the instantaneous rotation centers of indeterminate planar linkages," *Robotics*, vol. 11, no. 1, p. 6, 2021. https://doi.org/10.3390/robotics11010006 ↑ 3

[5] R. Di Gregorio, "An algorithm for analytically calculating the positions of the secondary instant centers of indeterminate linkages," *J. Mech. Des.*, vol. 130, no. 4, p. 042302, 2008. https://doi.org/10.1115/1.2839008 ↑ 3

[6] Y. P. Chang and I. Her, "A virtual cam method for locating instant centers of kinematically indeterminate linkages," *J. Mech. Des.*, vol. 130, no. 6, p. 062301, 2008. https://doi.org/10.1115/1.2900720 ↑ 3

[7] J. I. Valderrama-Rodríguez, J. M. Rico, and J. J. Cervantes-Sánchez, "A general method for the determi-nation of the instantaneous screw axes of one-degree-of-freedom spatial mechanisms," *Mech. Sci.*, vol. 11, no. 1, pp. 91–99, 2020. https://doi.org/10.5194/ms-11-91-2020 ↑ 3

[8] J. G. De Jalón, J. Unda, and A. Avello, "Natural coordinates for the computer analysis of multibody sys-tems," *Comput. Methods Appl. Mech. Eng.*, vol. 56, no. 3, pp. 309–327, 1986. https://doi.org/10.1016/0045-7825(86)90044-7 ↑ 4

[9] J. G. De Jalón and E. Bayo, *Kinematic and Dynamic Simulation of Multibody Systems*. Berlin, Germany: Springer-Verlag, 1994. ↑ 4, 11

[10] N. N. Romero, "Contributions to the kinematics and balancing of mechanisms," Ph.D. dissertation, 2023. ↑ 4

[11] R. Sancibrian, E. G. Sarabia, A. Sedano, and J. M. Blanco, "A general method for the optimal synthesis of mechanisms using prescribed instant center positions," *Appl. Math. Model.*, vol. 40, no. 3, pp. 2206–2222, 2016. https://doi.org/10.1016/j.apm.2015.09.032 ↑ 4

[12] J.-F. F. Collard, "Geometrical and kinematic optimization of closed-loop multibody systems," Ph.D. dis-sertation, Université Catholique de Louvain, 2007. ↑ 8

[13] H. R. Cazangi, "Aplicação do método de Davies para análise cinemática e estática de mecanismos com múltiplos graus de liberdade," M.S. thesis, 2008. ↑ 11

[14] R. L. Norton, *Design of Machinery: An Introduction to the Synthesis and Analysis of Mechanisms and Machines*. New York, NY, USA: McGraw-Hill Higher Education, 2003. ↑ 4, 11

[15] N. N. Romero Nuñez, "Simulation of a four-bar mechanism in MATLAB and GIM," *Zenodo*, 2026. https://doi.org/10.5281/zenodo.19378994 ↑ 12

[16] N. N. Romero Nuñez, "Simulation of a six-bar mechanism in MATLAB and GIM," *Zenodo*, 2026. https://doi.org/10.5281/zenodo.19379264 ↑ 12