

Optimal capacitor placement for distribution systems using genetic algorithms

Ubicación óptima de capacidades para sistemas de distribución usando algoritmos genéticos

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Clasificación del artículo: Investigación.

Fecha de aceptación: junio 30 de 2004

Fecha de recepción: septiembre 5 de 2003.

Key words: compensation methods for radial systems, genetic algorithms, stochastic load flows

Palabras clave: métodos de compensación para sistemas radiales, algoritmos genéticos, flujos de carga estocásticos.

ABSTRACT

In this paper we describe a new technique for placing fixed capacitors in radial distribution systems based on genetic algorithms (GA). Actually, current optimization models of capacitor placement only consider losses reduction and voltage profile simultaneously, but the compensation cost and the load changes are not taking into account as part of the objective function. Also, the result may be not the best choice because this is a very large optimization problem and there are too many combinations. We present here a general approach for the optimal solution of this problem considering all the parameters of the distribution system involved: capacitor cost, voltage, angle and load changes per hour. An exhaustive search through all possible solutions is needed. Therefore, GA is an ideal candidate to solve this situation. Working with this algorithm, a Microsoft Excel program has been developed in order to answer the capacitor placement problem for any kind of radial distribution system. The test results are presented along with the discussion of the algorithm.

RESUMEN

En este artículo se describe una nueva técnica para localizar condensadores fijos en sistemas radiales de distribución utilizando algoritmos genéticos (AG). Actualmente, los métodos de optimización corrientes sólo consideran, en forma simultánea, la reducción de pérdidas y el perfil de voltaje; no obstante, el costo de la compensación y las variaciones en la carga no son tomados en cuenta como parte de la función objetivo. Además, el resultado puede no ser el óptimo, debido a que éste es un gran problema de optimización y existen demasiadas combinaciones. Aquí se presenta una aproximación general para la solución óptima del problema, considerando todos los parámetros involucrados en un sistema de distribución: costo de condensadores, voltaje, ángulo y variaciones de carga por hora. Dada la necesidad de una búsqueda exhaustiva alrededor del espacio solución, los algoritmos genéticos son una opción ideal para resolver esta dificultad. Utilizando este concepto se ha desarrollado un programa en Microsoft Excel, con el objetivo de solucionar el problema de compensación reactiva para cualquier esquema de configuración de sistemas radiales de distribución. Los resultados de los circuitos ejemplo son presentados junto con la descripción del algoritmo.

1. Introduction

One of the usual techniques to reduce energy losses for radial distribution systems is the location of capacitors on one or more nodes of the system (Aoki, Kuwabara, Sato, 1988, 1.865-1.872). They are a source of reactive power in order to improve voltage profile of each one of the nodes and to increase energy passing through the lines. Also the substation could be able to provide more active power to the consumers because the inductive currents become smaller.

An ideal reactive compensation requires that all the inductive currents be zero, but this just can be possible placing capacitors of the required kVAr on every node, and it is impractical due to the excessive cost and the load variation. The model for the optimal capacitor placement is a combinatorial problem, because it is necessary to consider all the possible choices generated due to the location of m capacitors with x sizes on n nodes. Solution must be one which reduces the most possible energy losses and needs the less money investment considering maintenance and capacitors costs.

To achieve this, we have designed a model where the objective function to be minimized includes total active losses of the network together with investment cost. System's losses are the result from the load flow technique used. In this project, we used the "Fast decoupled Newton-Raphson method" (Stott, 1972, 1.955-1.972). Based on it, power and energy losses were determined by some example systems simulations. After it, optimal capacitor placement was developed by use of Genetic Algorithms (GAs). They are used to solve a great number of very important applications as massive searching problems in combinatorial optimization, operations research and numerical computation (Aoki, Kuwabara, Kanezashi, Sato, 1988, 1.267-1.274). GAs are the key to find the optimal solution for these types of problems because they are evolutionary algorithms very well known to be robust methods due to their ability for exploring all possible combinations on the solutions set and find the optimal in a reasonable amount of computational time.

In this paper, GA has been successfully tested on various radial distribution systems. The model proposed was implemented on a *Microsoft Excel* program. The rest of the paper is organized as follows. In section 2, we describe fast decoupled Newton-Raphson method, placing it in the context of existing models for radial distribution systems. In section 3, we provide the structure of genetic algorithms. We then show an example for optimal reactive compensation using the software in section 4. In section

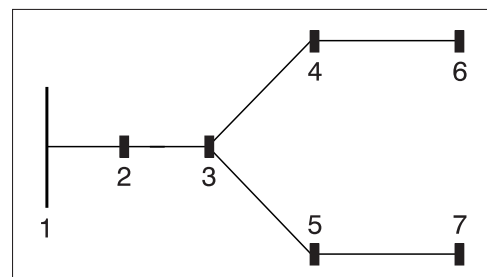
5, we briefly explain the implementation of the test circuit and the results, and we conclude in section 6 with a brief discussion and suggestions for further work.

2. Load flow

The mainly parameters for a load flow analysis are: voltage, angle, active and reactive power for each one of the circuit nodes and power system's losses. There are several methods to determinate those (Conner, 1973), but in the case of radial distribution systems, the most of these models do not consider all the variables involved in the problem or the time in computational resources is too high and thus, impractical. In example, in Gauss-Seidel model only the node voltage is regarded and not the angle, also there is a convergence problem if the starting value is not adequate (Dommel *et al.*, 1968, 1.866 -1.876). Under these restrictions the load flow analysis was developed by means of fast decoupled Newton-Raphson method. It solves the problem in a reasonable amount of computational time getting detailed information about interesting variables. Voltage and angle per node, such as active and reactive power are answers from this algorithm. The only constrain is that it requires a great numerical relation between reactive (X) and real part (R) from the conductors impedance, but this inconvenience can be solved artificially increasing the X/R ratio by use of negative reactances.

2.1 Newton-Raphson method for radial systems

The aim of this algorithm is to find voltage and angle values for every node, given active and reactive power for each one. Also, we know the feeder voltage (in our case, we take this value as 11.4 kV) and the conductors impedance. Power system's losses can be expressed as the difference between generated active power (node 1) and active power demanded by the other nodes. Figure 1, allows us to see a typical radial system problem (all the branches impedances were taking as $0,1609 + j 0,3847$ ohms per km).



Node i	Voltage (kV)	Angle (degree)	Act. Power Demanded Pd (kW)	React. Power Demanded Qd (kVAr)
1	11.4	0	?	?
2	?	?	385	288.75
3	?	?	485	363.66
4	?	?	302.5	226.82
5	?	?	170	127.47
6	?	?	86	54.85
7	?	?	230	137

Figure 1. Radial system problem formulation

First of all, every voltage and power must be converted into per unit system. Then, 11.4 kV is considered as 1. This is base voltage. Also we need to define base power in MVA. In this case, we take it as 1 MVA. Thus, for consistency, base impedance must be (1):

$$\text{Base impedance (ohms)} = \frac{\text{Voltage Base (kV)}^2}{\text{Apparent Power Base (MVA)}} \quad (1)$$

A per unit value is developed as real value divided by base value, in a way described below (2):

$$\text{Per unit value} = \frac{\text{Real Value}}{\text{Base Value}} \quad (2)$$

Now, we present the steps procedure to evaluate the algorithm:

- *Step 1.* Build the admittance matrix Y taking admittance values of each one of the branches from node I to node J . G_{ij} is a real value and B_{ij} is an imaginary value from the matrix. It has the $n \times n$ structure, where n is the number of nodes.
- *Step 2.* Keep only imaginary values of the matrix, B_{ij} and build with them a new matrix called B'' .
- *Step 3.* Solve equations (3), (5) until the errors $\Delta Pn'$ and $\Delta Qn'$ will be less than a tolerance ϵ . This tolerance should be very close to zero, i.e. $1 \cdot 10^{-4}$. Increase angle and voltage values in $\Delta\theta$ (4) and ΔE (6). 1 and 0 are considered as initial values of voltage and angles, respectively. Repeat this iterative process until tolerance will be satisfied.

$$-B'' \Delta\theta = \Delta Pn' \quad (3)$$

$$\theta = \theta_i + \Delta\theta \quad (4)$$

$$-B'' \Delta E = \Delta Qn' \quad (5)$$

$$E = E_i + \Delta E \quad (6)$$

$\Delta Pn'$ and $\Delta Qn'$ denote errors of active and reactive power from node 2 to n . They depend of the admittance matrix parameters, just as, voltage and angle values for each node. We have already calculated some of these values in step 1 and then we can consider them as constants. The others (E, θ) are variables and should be fixed until the tolerance has a right value. Expressions (7) and (8) show how these errors are calculated for nodes $i = 2$ to n :

$$\Delta Pn'_i = \frac{\left(-Pd_i - \sum_{m=1}^n E_i E_m (G_{im} \cos(\theta_i - \theta_m) + B_{im} \sin(\theta_i - \theta_m)) \right)}{E_i} \quad (7)$$

$$\Delta Qn'_i = \frac{\left(-Qd_i - \sum_{m=1}^n E_i E_m (G_{im} \sin(\theta_i - \theta_m) - B_{im} \cos(\theta_i - \theta_m)) \right)}{E_i} \quad (8)$$

Finally, we have the correct voltage and angle for every node. Flow chart of Fast decoupled Newton-Raphson method is shown in figure 2. We are now ready to evaluate total active loss of the network. It is expressed as (9):

$$\sum_{m=1}^n E_1 E_m (G_{1m} \cos(\theta_1 - \theta_m) + B_{1m} \sin(\theta_1 - \theta_m)) - \sum_{i=2}^n Pd_i \quad (9)$$

Where E_1, θ_1 are voltage and angle for reference node 1 (1,0), and "n" denotes number of system's nodes. The first summation represents total active power given for the radial feeder and the second represents total active power demanded for the other nodes. Thus, the difference between them gives us total active power losses on the branches.

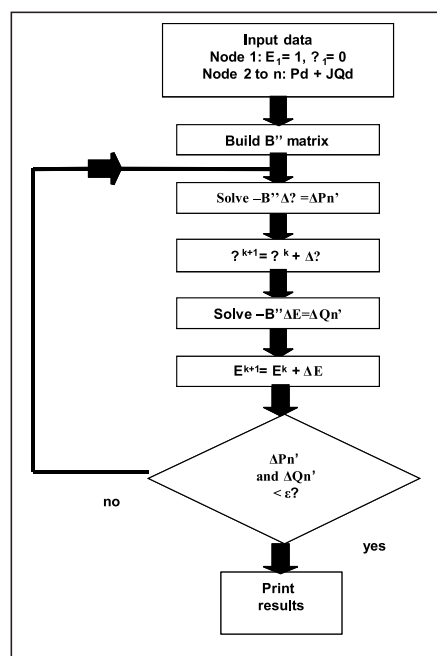


Figure 2. Flow chart of Fast decoupled Newton-Raphson Method

3. A genetic algorithm approach

In this section, the implementation of genetic algorithm is discussed. GA is a search algorithm based on the mechanic of natural selection. Basically, a GA makes a population that evolve through time using reproduction and mutation process (Nara *et al.*, 1992:1.044-1.051). Only individuals representing good solutions of the capacitor placement problem will survive longer, and their genetic information will be present in the next generation. At the end, after several generations, the interaction between these high quality individuals will produce a final population which represent the best solutions set of the problem.

3.1 Fixed capacitor encoding

Before of the genetic algorithm procedure, the real parameters of the problem must be represented in genetic algorithm language. It means that location and size of the capacitors used are codified as a chromosome. The representation chosen for this application is a chromosome divided in two parts (Mendes *et al.*, 2001). First part indexes location of the capacitors. It is represented as binary values, where 1 indicates the presence of one capacitor on a specific node and 0 the absence. The second part indicates the size of the capacitors used. It is composed by integer values, where each one symbolizes one type of capacitor size in kVAr. Figure 3 and Table 1 show a simple schematic diagram of a chromosome encoding for the problem of Figure 1.

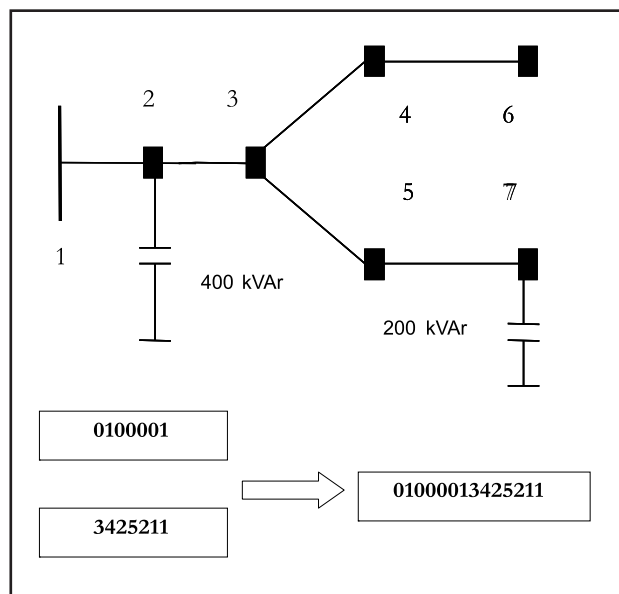


Figure 3. Capacitor placement codified as a chromosome

Table 1. Capacitor sizes

Type	Size (kVAr)
1	200
2	400
3	600
4	800
5	1.000

It is possible to see that nodes 2 and 7 have capacitors of 400 kVAr (type 2) and 200 kVAr (type 7). The binary chromosome structure has the value equals 1 in second and seventh position. For integer part, the sequence that follows is indifferent because the other values of the first part of the chromosome are 0. In such cases, the numbers of the second part should be ignored.

3.2 Algorithm methodology

The overall procedure is summarized as follows:

- 1) Create the genetic reserve at random
- 2) For each chromosome, evaluate objective function and determine if technical restrictions are satisfied. If chromosomes population have converged, capacitor results are printed. Unless go to the next step
- 3) Recombine individuals based on reproduction mechanism
- 4) Mutate some of new individuals (10%)
- 5) Evaluate objective function with individuals from the previous step. Go to step 2.

Flow chart of solving the fixed capacitor placement problem in distribution system is shown in Figure 4. The main parameters to determinate if a chromosome is a very good solution depend of two factors; the objective function value and the technical restrictions.

The objective function includes a sum of capacitors and system's losses costs (15 years). The less value subject to restrictions must be found by GA.

$$Objective\ Function = 8760 \times System's\ losses\ per\ hour\ (Kwh) \times energy\ cost\ (USD\$/Kwh) \times 15 + Capacitor\ cost\ (USD\$) \quad (10)$$

We have considered 15 years as duty life of capacitors. It is the capital recovery period respect to the initial cost of capacitors (Merlin & Back, 1975). Energy cost used was the sum of generation and transmission costs.

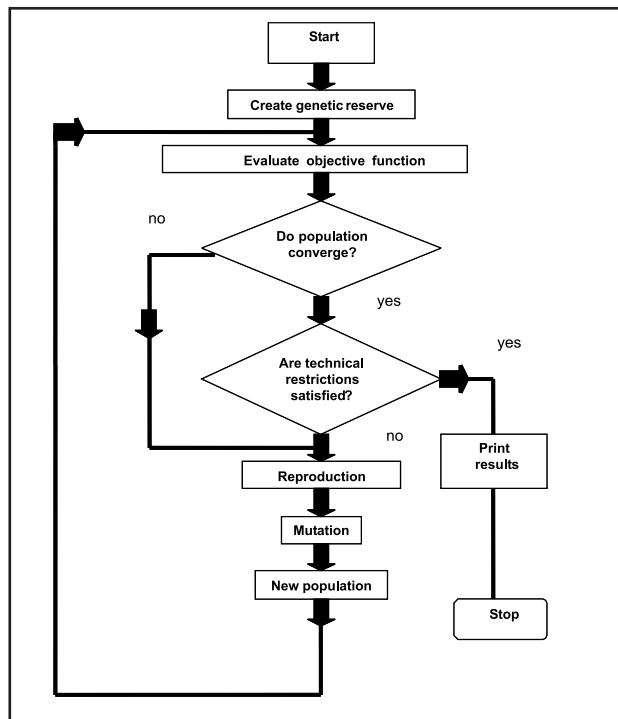


Figure 4. Flow chart of the GA used to solve the capacitor placement problem.

Technical restrictions embrace all possible solutions that even having a good value for objective function involve an unacceptable increase or decrease of some parameters of the system. These parameters are:

- Magnitudes of the voltage on system’s nodes: nodal voltages must be in a range between 0.98 and 1.02 pu. It means that for all transformers connected to the nodes will be at least a minimal value that assures a good voltage value in secondary network. This range can be fixed if the system does not require a tolerance too close respect to feeder voltage.
- Current flow by conductors: given a capacitor placement structure by GA, none of the branches current circulating around system can be higher than maximal current supported by conductors.
- Maximal reactive power provided by capacitors: let’s total reactive power demand of the distribution system be QT. Then, the sum of every capacitor connected on the nodes can not be higher than QT. It implies that never will be a current flowing toward feeder at peak load.

We now describe briefly how the mechanisms of reproduction and mutation are simulated by GA. In reproduction process, firstly we randomly select a pair of chromosomes, with same structure as we have shown in Figure 3. In the next step, chromosomes are treated separately; one for binary part and another for integer part. In binary part, for a given position, if two parents share value, the chromosome produced by reproduction will keep it. If values are different, the result for new chromosome is selected at random. In integer part, for a given position, result will be the average of values found in the parents. If result is not an integer value it will be approximated until closer value at random. In Figure 5, two chromosomes bind together forming a new chromosome under reproduction mechanism.

$$\begin{array}{r}
 11100013241521 \\
 + \\
 01110012143212 \\
 = \\
 01110013242421
 \end{array}$$

Figure 5. Reproduction process between two chromosomes

In mutation process, chromosome structure is modified. This change is performed at random, but there is a difference between binary and integer part. First one, is modified by choosing a position of individual at random and changing the value for its opposite (1 by 0 and 0 by 1). Second part acts on integer values by adding or subtracting an unity from its value. For the chromosome shown in Figure 6, its structure is different after mutation process.

$$\begin{array}{r}
 11100013241521 \\
 + \\
 \text{MUTATION} \\
 = \\
 10001014132412
 \end{array}$$

Figure 6. An example of a chromosome produced after mutation process

4. Computational Application

4.1 Load flow

Load flow method exposed above was developed using VB for Applications MS Excel. This flexible tool allows

set parameters as topology, current capacity, reactance, resistance, susceptance, gauge and material of conductors, variable load, energy cost, branch length, tolerance, base power, base voltage, and others.

We now describe how the model detailed in section 1 (example of Figure 1), may be simulated using this program. We first describe how we can fill the program columns. For each conductor that has different value from impedance and susceptance we write the type used in the parameters sheet. Simultaneously, we write active and power demand for a twenty four hours period. In our example, we are working with one conductor (397.5 MCM) and with constant load for each hour of the entire day. Figure 7, shows this sheet applied to our studied case.

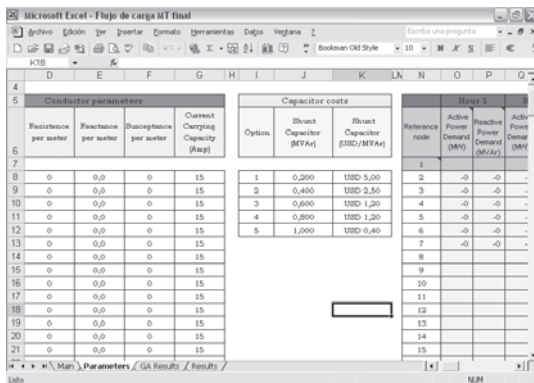


Figure 7. Input parameters for a radial distribution system

After it, we should describe the topology of the circuit in main sheet; it means that for each branch of a circuit we must write the kind of conductor used and its length. Also, base voltage (feeder output), base power, such as maximal admissible tolerance for the iterative process; make part of the information needed. In the example case, we take all branches distances as 1 km and base power as 1 MVA (see Figure 8).

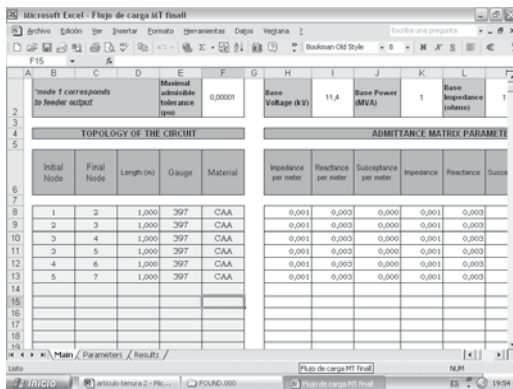


Figure 8. Topology description of radial system

Finally, we can simulate the behavior of a circuit given. Figure 9, shows program results. In *main* sheet, we can see voltage and angle for every node as real value. At the same time, we can visualize total system's losses (MW). In *results* sheet, system losses (MW) and Maximal drop of voltage are shown for each one of twenty four hours. In our example case, these losses take same value in per unit and in real value because the first has been consider as 1 MVA. From analysis made to example system, it is possible to conclude that system losses in one day were 0.21528 MWh and all the voltages are very close to 11.4 kV, minimal was 11.263 kV (node 7) and maximal was 11.335 kV (node 2). Then, it is possible to conclude that this configuration system is working under normal operating conditions because a slow variation rate of the node voltages suggests a minimal value of system's losses.

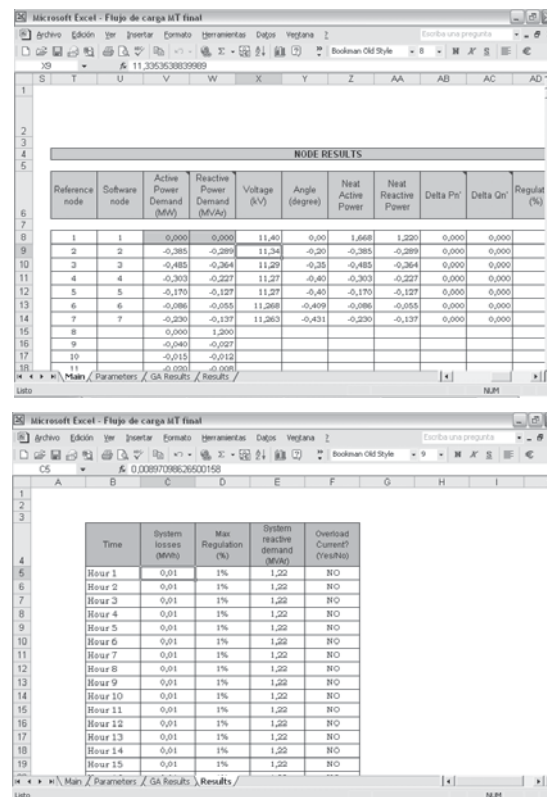


Figure 9. Final software results in example problem of Figure 1

4.2 Genetic algorithms

Methodology exposed in section 3, is implemented in software. All parameters used in the algorithm are flexible and they can be modified. These parameters are: population, combination rate, mutation rate, energy cost and number of generations.

Population refers to all the initial chromosomes selected at random and encoding some of alternatives to solve capacitor placement problem. Combination rate means what is the increase of population percent respect to initial population; in example, if initial population is 10, after reproduction process, new population will be 13. Mutation rate is the percent of chromosomes to be modified under this process depicted in last section. Energy cost is the price of losses expressed in its monetary equivalent. Units used were dollars per MWh (US\$/MWh). Numbers of generations are the quantity of new populations generated under flow chart exposed in section 3.

Once these parameters have been fixed, GA runs and gives us an answer. It puts all the different combinations of chromosomes produced during generations. We must select chromosomes which have the less value possible of objective function. In the program it corresponds to the column (total USD). At right, we can see the fixed capacitor assigned to each one of the nodes.

The menu can be seen in Figure 10.

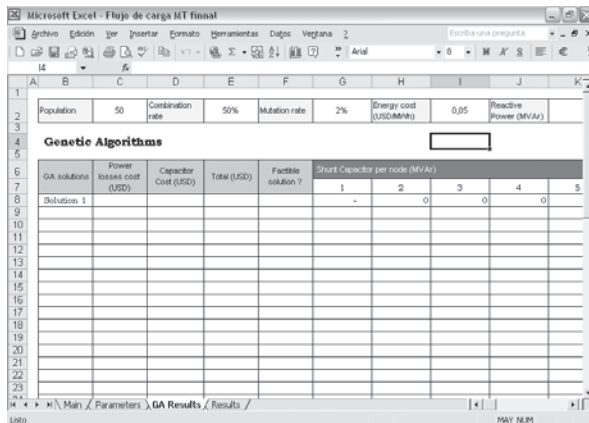


Figure 10. GA menu for the capacitor placement software designed

5. Test case

Proposed method was implemented in software to find the optimal capacitor placement for topology of Figure 1 with variable load (see Figure 11) on each one of the 6 nodes during a twenty four hours period.

Parameters of simulation were: base voltage 11.4 kV, base power 1 MVA, maximal admissible tolerance for load flow error 0.00001, maximal admissible tolerance for regulation (increase or decrease of node voltage respect feeder voltage) 5% and energy cost 20.5 US\$/MWh. Also, capacitors cost was listed as we can see in Table 2.

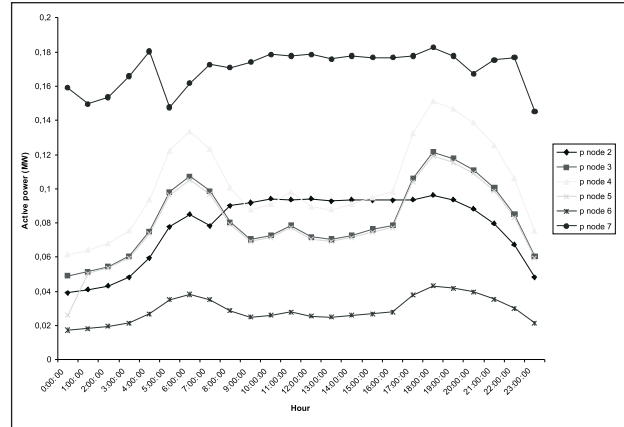


Figure 11. Load variation for active power used in simulation of test case

Table 2. Capacitors cost used for the simulation case

Option	Shunt capacitor (MVar)	Cost (US\$/MVar)
1	0.2	5000
2	0.4	2500
3	0.6	1200
4	0.8	1200
5	1	400

The results of energy losses for a period of 15 years considering the cost exposed above without compensation were 1.736.265 dollars. Solutions given by GA are listed in Figure 12.

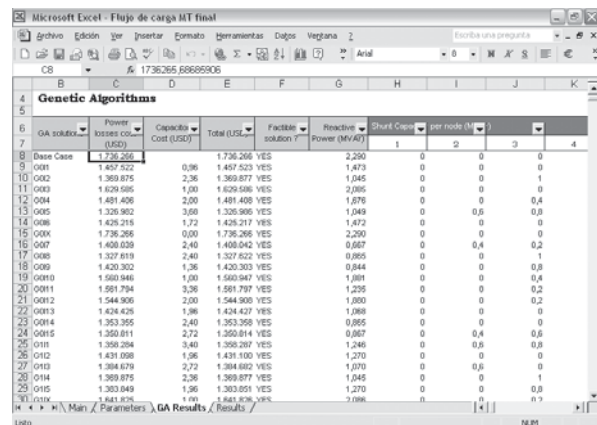


Figure 12. List of chromosomes generated for the test case

These results are the different chromosomes produced in order to solve capacitor placement in test case. There are good solutions that produce a drastic

reduction of energy losses. Best of these one was the capacitors location with sizes of 800 kVAr, 400 kVAr, 400 kVAr on the nodes 3, 5 and 6, respectively. It has a decrease of 23.77% in energy losses respect to the case without compensation. In monetary terms it means that objective function has the value of 1.323.490 U\$ versus 1.736.265 U\$. It implies a significant economic saving, because even including capacitor investment the objective function has a less value than cost of energy losses without compensation.

6. Conclusions

In this paper, a new method for solving capacitor placement problem was described. It can provide specific location of fixed capacitors in order to reduce energy system's losses. The load variation, the energy cost and capacitors sizes easily found in market were considered in the model.

From analysis made to test network, it is possible to conclude that the method works satisfactorily, revealing the convenience of selective capacitor placement, due to the drastically reduction of energy losses, 24 % for our example.

The program developed can be used in radial systems with different topologies and load variation because it has flexible parameters. Also it can be used as an analysis tool to make planning studies or to take decisions about the convenience of a specific reactive compensation plan. Then, is a powerful tool in the design of an electric distribution system.

This method has a better performance than an expert system because it can explore more than 100.000 solutions in a very short computational time. Also the technical restrictions imposed in the model exclude un-factible solutions. So, the work presented in this paper is possible, cheap and reliable to find the optimal capacitor placement in a radial system.

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