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Artículo de revisión

Series and Parallel Connections: An Articulating Topic in Teaching Physics

Conexiones en serie y en paralelo: un tema articulador en la enseñanza de la física

Conexões em série e em paralelo: um tema articulador no ensino da física

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ABSTRACT

The fragmented treatment of series and parallel connections across isolated chapters in university physics textbooks limits students' ability to recognize the deep conceptual structure shared by these configurations. To address this issue, we conducted a systematic review of 11 widely used textbooks, peer-reviewed literature, and large language models (ChatGPT and DeepSeek), identifying 15 physical devices that admit series and parallel configurations across multiple physics domains: electromagnetism, oscillations and waves, fluid mechanics, thermodynamics, geometrical optics, and classical mechanics. We propose treating these connections as a unifying theme in university-level physics education, grounded in four fundamental conservation principles: energy, mass, charge, and linear momentum. To this effect, we present a constructivist instructional sequence, structured around the POSSE methodology and PhET interactive simulations, designed

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for introductory and intermediate physics courses without the need for curricular restructuring. This integrative approach allows students to transfer a single conceptual framework across physics domains, replacing fragmented procedural learning with meaningful cross-domain understanding. Additionally, we derive a new general expression for the equivalent mass of *n + 1* identical masses suspended from *n* pulleys in series, extending previous treatments beyond low-order cases. We also demonstrate that it is impossible for any pair of physically admissible devices to yield the same equivalent quantity in both series and parallel configurations, reflecting the fundamentally distinct algebraic structures for each connection type.

Keywords: series connections, parallel connections, conservation principles, physics education

RESUMEN

El tratamiento fragmentado de las conexiones en serie y en paralelo a lo largo de capítulos aislados en los libros de texto universitarios de física limita la capacidad de los estudiantes para reconocer la profunda estructura conceptual que comparten estas configuraciones. Para abordar este problema, llevamos a cabo una revisión sistemática de 11 libros de texto ampliamente utilizados, literatura revisada por pares y modelos de lenguaje extensos (ChatGPT y DeepSeek), identificando 15 dispositivos físicos que admiten configuraciones en serie y en paralelo a través de múltiples dominios de la física: electromagnetismo, oscilaciones y ondas, mecánica de fluidos, termodinámica, óptica geométrica y mecánica clásica. Proponemos tratar estas conexiones como un tema unificador en la educación en física a nivel universitario, con fundamento en cuatro principios de conservación fundamentales: energía, masa, carga y momento lineal. Para ello, presentamos una secuencia instruccional constructivista, estructurada en torno a la metodología POSSE y simulaciones interactivas PhET, diseñada para cursos de física introductorios e intermedios sin requerir una reestructuración curricular. Este enfoque integrador permite a los estudiantes transferir un único marco conceptual a través de los dominios de la física, reemplazando el aprendizaje procedimental fragmentado por una comprensión transversal y significativa. Adicionalmente, deducimos una nueva expresión general para la masa equivalente de *n + 1* masas idénticas suspendidas de *n* poleas en serie, extendiendo los tratamientos previos más allá de los casos de orden inferior.

También demostramos que es imposible que cualquier par de dispositivos físicamente admisibles produzca la misma cantidad equivalente tanto en serie como en paralelo, lo que refleja las estructuras algebraicas fundamentalmente distintas de cada tipo de conexión."

Palabras clave: conexiones en serie, conexiones en paralelo, principios de conservación, enseñanza de la física

RESUMO

O tratamento fragmentado das conexões em série e em paralelo em capítulos isolados dos livros didáticos de física universitária limita a capacidade dos estudantes de reconhecer a estrutura conceitual profunda compartilhada por essas configurações. Para abordar essa questão, realizamos uma revisão sistemática de 11 livros didáticos amplamente utilizados, da literatura revisada por pares e de modelos de linguagem de grande porte (ChatGPT e DeepSeek), identificando 15 dispositivos físicos que admitem configurações em série e em paralelo em múltiplos domínios da física: eletromagnetismo, oscilações e ondas, mecânica dos fluidos, termodinâmica, óptica geométrica e mecânica clássica. Propomos tratar essas conexões como um tema unificador no ensino de física em nível universitário, fundamentado em quatro princípios de conservação fundamentais: energia, massa, carga e momento linear. Para tanto, apresentamos uma sequência instrucional construtivista, estruturada em torno da metodologia POSSE e de simulações interativas do PhET, projetada para cursos introdutórios e intermediários de física sem a necessidade de reestruturação curricular. Essa abordagem integrativa permite que os estudantes transfiram um único referencial conceitual entre domínios da física, substituindo a aprendizagem procedimental fragmentada por uma compreensão significativa entre domínios. Além disso, derivamos uma nova expressão geral para a massa equivalente de n + 1 massas idênticas suspensas em n polias em série, estendendo tratamentos anteriores para além dos casos de baixa ordem. Também demonstramos que é impossível que qualquer par de dispositivos fisicamente admissíveis produza a mesma grandeza equivalente tanto na configuração em série quanto na em paralelo, refletindo as estruturas algébricas fundamentalmente distintas de cada tipo de conexão.

Palavras-chave: conexões em serie, conexões em paralelo, princípios de conservação, ensino da física

INTRODUCTION

One significant challenge stemming from traditional teaching approaches in introductory and intermediate university physics courses is the lack of articulation between courses and their contents. From this perspective, students tend to perceive each physics course as a disconnected or isolated set of topics, which generates fragmented knowledge instead of favoring a comprehensive understanding of physics. Several authors have studied this problem. For example, Kaltakci-Gurel *et al.* (2016) reported that knowledge becomes fragmented in physics teachers in training when studying spherical mirrors and lenses. In turn, Hestenes (1997) proposes a methodology that allows teachers in training to develop a more structured scientific knowledge of physics, in contrast to the fragmentation caused by traditional instruction. Additionally, Solarte (2006) discusses the fragmentation of scientific concepts and their incorporation into pedagogical models in textbooks, specifically analyzing the concept of *energy*.

From the perspective of meaningful learning, this fragmentation is not only a curricular problem; it is also a cognitive one. According to Ausubel (2002), meaningful learning occurs when students relate new concepts to their previous knowledge. In this type of learning, knowledge integration becomes a substantial, non-arbitrary cognitive structure with meaning and understanding. Particularly, Moreira (2021) states that one of the challenges in physics instruction is achieving meaningful learning among students, thereby avoiding the superficial accumulation of unconnected facts and procedures. Similarly, Wieman *et al.* (2008), emphasize that students learn most effectively when they construct their own understanding of scientific ideas within the framework of their existing knowledge.

When physics content is presented as a series of isolated chapters, students are driven towards rote learning. They acquire formulas and definitions that are never subsumed under more inclusive concepts, so their knowledge remains unstable, difficult to retrieve, and hard to transfer to new situations. This issue has been documented, for example, in students' learning of Newton's third law (Bao & Fritchman, 2021), simple electric circuits (Liu *et al.*, 2022), work and mechanical energy (Tong *et al.*, 2023), mechanical wave propagation (Xie *et al.*, 2021), and electromagnetic induction (Xie *et al.*, 2025).

Based on our university-level teaching experience in fundamental classical physics courses (electromagnetism, oscillations and waves, fluids, thermodynamics, geometric optics, and classical mechanics), we have observed that, in some well-known university physics textbooks (Alonso & Finn, 1983; Freedman & Young, 2010; Giancoli, 2013; Halliday *et al.*, 2021; Hecht, 2017; Ogata, 2003; Ohanian & Markert, 2007; Sadiku, 2006; Serway & Jewett, 2015; Tipler & Mosca, 2007; Tippens *et al.*, 2007), series and parallel connections in different areas of physics are addressed as isolated issues. Sometimes, they are presented as subsections of chapters, as examples, or as proposed problems. In some of these textbooks, the corresponding formulas are presented only in the chapter summary.

The main objective of this work is to show that series and parallel connections can be presented as an integrative topic that articulates and relates several physics areas through a fundamental aspect: the conservation principles of energy (CPE), charge (CPC), mass (CPM), and linear momentum (CPP).

This proposal has been designed for both introductory and intermediate levels, and it does not require a structural modification of the curriculum. It is best implemented in curricula where conservation principles are central learning outcomes and where teachers seek to reduce conceptual fragmentation by establishing explicit cross-domain connections between electromagnetism, oscillations and waves, fluids, thermodynamics, geometric optics, and classical mechanics. The integration advocated herein is conceptual: series and parallel connections are treated as an articulating theme that can be revisited across different courses, making explicit cross-domain links when each topic is addressed in its corresponding curricular context.

In this research, we conducted a comprehensive literature review on examples of series and parallel connections using university textbooks, Google Scholar, and artificial intelligence (AI) tools such as ChatGPT and DeepSeek. We identified 14 examples, which were distributed as follows: five in electromagnetism, four in oscillations and waves, two in fluid mechanics, one in thermodynamics, one in classical mechanics, and two in geometric optics. In addition, we specifically analyzed series and parallel connections of Atwood machines and examined the possibility of two devices connected in series or parallel producing the same equivalent physical quantity. Moreover, we propose a teaching-learning sequence grounded in the POSSE methodology to implement the integrated study of series and parallel connections.

This article is organized as follows. The second section describes the methodology used in the search for series and parallel connections, including the instruments and procedures employed, and the third section presents the results obtained. Finally, the fourth section presents the main conclusions of this study. The supplementary material contains the teaching-learning sequence.

METHODOLOGY

We searched for series and parallel connections in physics using university physics textbooks, the Google Scholar search engine, and two large language models (LLMs): DeepSeek (2025) and ChatGPT (2025).

Phase 1: textbook review

In this phase, we reviewed 11 textbooks (Alonso & Finn, 1983; Freedman & Young, 2010; Giancoli, 2013; Halliday *et al.*, 2021; Hecht, 2017; Ogata, 2003; Ohanian & Markert, 2007; Sadiku, 2006; Serway & Jewett, 2015; Tipler & Mosca, 2007; Tippens *et al.*, 2007) that we have used as physics professors at the university level. These textbooks were selected based on explicit academic and curricular criteria. All of them have been widely adopted in undergraduate physics and engineering programs and have been published in multiple editions, which reflects their sustained relevance and pedagogical refinement. Their broad use ensures that our analysis addresses instructional materials that shape the learning experiences of a large and diverse student population.

The textbooks provide comprehensive coverage of core physics domains, including mechanics, electromagnetism, waves, fluids, thermodynamics, and geometric optics, and they span both introductory and intermediate levels.

In this stage of the study, we identified 14 examples of series and parallel connections distributed across five physics areas. These connections are listed by area.

- *Electromagnetism:* series and parallel connections of resistors, inductors, capacitors, capacitors with dielectrics, and cables.
- *Fluids:* series and parallel connections of pipes (for liquids and gases) and centrifugal pumps.
- *Thermodynamics:* series and parallel connections of walls of different materials that allow heat flow.
- *Oscillations and waves:* series and parallel connections of springs, speakers, bungee cords, and dampers.
- *Geometric optics:* series and parallel connections of lenses and mirrors.

Phase 2: Google Scholar search

Subsequently, we searched Google Scholar via the instruction "series and parallel connections". In addition to the connections identified in the first phase, we found series and parallel connections of Atwood machines (Matehkolaee, 2011) and centrifugal pumps (which had already been identified in the previous phase). Consequently, this second search phase yielded only one new series and parallel connection in the field of classical mechanics.

Phase 3: LLM search

LLMs, specifically ChatGPT and DeepSeek, were used as a complementary exploratory tool to identify potential physical systems that admit series and parallel configurations across different areas of physics. We conducted searches in DeepSeek and ChatGPT using the following prompt: "generate a list of physical systems organized in series and parallel configurations. For each system, include its characteristic formula and the applicable conservation principle".

During the LLM search phase, we prompted the models to generate lists ranging from 20 to 100 examples of physical systems that can be arranged in series and in parallel configurations. The output was subsequently examined manually and analyzed item by item. Each example was cross-checked against the list previously compiled from university textbooks, and the corresponding expressions provided for the equivalent physical quantity were carefully reviewed to verify their consistency with established formulations. This procedure allowed identifying redundancies, confirming valid cases, and assessing the reliability and scope of our LLM-generated results.

As a result, we obtained an extensive list of various series and parallel configurations across different physics branches, including electrical, mechanical, thermal, optical, and fluid dynamics systems, along with their applicable conservation principles (energy, charge, mass, and momentum). However, this list contained highly repetitive information and therefore did not provide series and parallel connections beyond those identified in the previous phases.

Repeated or overlapping cases were filtered using explicit exclusion criteria, and only distinct, verifiable configurations were retained for analysis. Although no fundamentally new examples emerged from this phase, it was still included in this work for two reasons: (i) it confirmed convergence between AI-generated suggestions and established disciplinary knowledge, and (ii) it provided evidence of the fragmentation observed in textbooks.

RESULTS

Table 1 summarizes the physical systems identified in this study. The table has four columns: the first lists the physical devices; the second contains the equivalent physics quantity, its symbol, and its units in the International System; and the third and fourth present the analytical expressions for the equivalent physics quantity in series and parallel connections, respectively.

In electromagnetic systems, R_{eq} denotes the equivalent electrical resistance measured in Ohms (Ω), G_{eq} the equivalent conductance in Siemens (S), L_{eq} the equivalent inductance in Henries (H), X_{eq} the equivalent reactance in Ohms (Ω), Z_{eq} the equivalent impedance in Ohms (Ω), and C_{eq} the equivalent capacitance in Farads (F). For capacitors with dielectrics, κ_i represents the dimensionless dielectric constant of the i-th layer.

In fluid systems, R_T denotes the hydraulic resistance, typically expressed in Pascal-seconds per cubic meter ($Pa \cdot s \cdot m^{-3}$); Q_i the volumetric flow rate in cubic meters per second ($m^3 \cdot s^{-1}$); and H_i the head supplied by the i-th pump, measured in meters (m).

In thermodynamic systems, $R_{T,eq}$ corresponds to the thermal resistance, in Kelvins per watt ($K \cdot W^{-1}$). In mechanical and oscillatory systems, k_{eq} represents the equivalent elastic constant in newtons per meter ($N \cdot m^{-1}$), and b_{eq} the equivalent damping coefficient in kilograms per second ($kg \cdot s^{-1}$). In geometric optics, f_{eq} denotes the equivalent focal length, measured in meters (m).

Each analytical expression is explicitly linked, in parentheses, to the conservation principle from which it is derived: CPE, CPC, CPM, or CPP. The purpose of Table 1 is therefore not merely to compile formulas, but to emphasize the shared conceptual structure underlying series and parallel connections across different areas of physics, reinforcing the role of the conservation laws as an articulating theme.

The following examples show how series and parallel connections can be studied using conservation principles:

Example 1 – capacitors in parallel. Applying the CPC, the charge supplied by the source equals the sum of the charges distributed across the plates of each capacitor, as expressed in Equation (1):

$$Q_{total} = \Sigma Q_i \quad i = 1,2,\ldots,n,. \tag{1}$$

Since the charge in each capacitor is given by $Q_i = C_i \cdot \Delta V$, substituting into (1) yields:

$$Q_{total} = \sum_{i=1}^{n} C_i . \Delta V$$

This expression can be rewritten in terms of an equivalent capacitance C_{eq}, as shown in Equation (2):

$$Q_{total} = C_{eq} \cdot \Delta V \tag{2}$$

where the equivalent capacitance is defined as

$$C_{eq} = \Sigma C_i \quad i = 1,2,\ldots,n. \tag{3}$$

Example 2 – series capacitors. The work supplied by the source when charging capacitors from Q = 0 to Q is given by Equation (4):

$$W_{source} = \int_0^Q \Delta V \, dQ = \int_0^Q \left(\sum_{i=1}^{n} \frac{Q}{C_i} \right) dQ = \frac{Q^2}{2} \left(\sum_{i=1}^{n} \frac{1}{C_i} \right) = \frac{Q^2}{2} \frac{1}{C_{eq}} \tag{4}$$

where the equivalent capacitance for capacitors in series is defined by Equation (5):

$$\frac{1}{C_{eq}} = \sum_{i=1}^{n} \frac{1}{C_i}. \tag{5}$$

On the other hand, the energy stored in each capacitor is given by Equation (6):

$$U_i = \frac{1}{2} C_i (\Delta V_i)^2 = \frac{Q^2}{2} \frac{1}{C_i}, \tag{6}$$

By summing the energy contributions of all capacitors, the total stored energy is obtained, as shown in Eq. (7):

$$U_{total} = \frac{Q^2}{2} \sum_{i=1}^{n} \frac{1}{C_i} = \frac{Q^2}{2} \frac{1}{C_{eq}}. \tag{7}$$

A comparison of (4) and (7) shows that $W_{source} = U_{total}$, validating the CPE and the derivation of equivalent capacitance from this principle.

Example 3 – blood vessels in parallel. Applying the CPM, the mass of blood per unit of time leaving a major artery must equal the sum of masses of blood per unit of time circulating through various blood vessels connected in parallel, assuming no accumulation or loss. For an incompressible fluid of constant density, such as blood, mass conservation translates to volumetric flow conservation $\left(Q = \frac{dV}{dt}\right)$:

$$Q_{total} = \sum_{i=1}^{n} Q_i, \quad (i = 1,2,\dots,n), \tag{8}$$

where Q_i is the flow in the *i*-th blood vessel.

The flow in each vessel can be modeled using a linear pressure-resistance relationship, as given in Equation (9):

$$Q_i = \frac{\Delta P}{R_i}, \tag{9}$$

where ΔP is the pressure difference between the ends of the vessel, and R_i is the hydrodynamic resistance of the *i*-th vessel.

Substituting (9) into (8) yields Equation (10):

$$Q_{total} = \sum_{i=1}^{n} \frac{\Delta P}{R_i}. \quad (10)$$

This expression can be rewritten in terms of the equivalent resistance, as shown in Equation (11):

$$Q_{total} = \frac{\Delta P}{R_{eq}}, \quad (11)$$

with the equivalent resistance of the parallel vessel system defined by Equation (12):

$$\frac{1}{R_{eq}} = \sum_{i=1}^{n} \frac{1}{R_i}. \quad (12)$$

Example 4 – serial blood vessels. Applying the CPM, the mass of blood flowing per unit of time must remain constant across the entire system of blood vessels connected in series, assuming no accumulation or loss. For an incompressible constant-density fluid, mass conservation means that the volumetric flow is the same in all vessels. The total pressure drop is given by Equation (13):

$$\Delta P_{total} = \sum_{i=1}^{n} \Delta P_i. \quad (13)$$

Since the pressure drop across each vessel is proportional to the flow rate, this expression can be rewritten as follows:

$$\Delta P_{total} = Q_{total} \sum_{i=1}^{n} R_i.$$

Defining an equivalent resistance, the total pressure drop can be expressed as

$$\Delta P_{total} = Q_{total} R_{eq},$$

where the equivalent resistance of the series system is defined by Equation (14):

$$R_{eq} = \sum_{i=1}^{n} R_i \,. \tag{14}$$

Therefore, the system can be represented as a single vessel with equivalent resistance R_{eq}.

Table 1

Equivalent physical quantities and conservation principles for series and parallel connections in different physical systems

Device	Equivalent quantity	Series connection	Parallel connection
Resistances	Resistance $R_{eq} (\Omega)$		
	Conductance Geq (S)		
Inductors	Inductance Leq (H)		
	Reactance Xeq (Ω)		
	Impedance Zeq (Ω)		
Capacitors	Capacitance Ceq (F)		
Capacitors with dielectrics	Capacitance Ceq (F)		
	Impedance Zeq (Ω)		
Cables	Resistance R_{eq} (Ω)		
Pipes (Blood vessels)	Hydraulic resistance $R_T (\mathrm{Pa \cdot s \cdot m^{-3}})$		
Hydraulic pumps	Overall efficiency η Dimensionless	$\eta_{SS} = \frac{\sum_{i=1}^{n} H_i}{\sum_{i=1}^{n} \frac{H_i}{\eta_i}}$. *(CPE)*	$\eta_{SP} = \frac{\sum_{i=1}^{n} Q_i}{\sum_{i=1}^{n} \frac{Q_i}{\eta_i}}$. (CPM)
Walls	Thermal resistance (K/W)		
Springs	Spring constant $k_{eq} (N/m)$		
Speakers	Impedance Zeq (Ω)		

Bungee cords	Spring constant $k_{eq}(N/m)$		
Dampers	Damping coefficient $c_{eq}(N.s/m)$		
Thin lenses	Focal length $f_{eq}(m)$		
Mirrors	Focal length $f_{eq}(m)$		

In Table 1, the series and parallel connections of the Atwood machines are not included, as they will be addressed separately.

Atwood machines in series and in parallel

Beyond the systems that are traditionally covered in university physics courses (Table 1), other devices are equally suited to series and parallel arrangements. A particular case is the Atwood machine, in which the concept of equivalent mass serves as the connecting quantity. For the simple Atwood machine, the equivalent mass is as follows (Matehkolaee, 2011):

Under this approach, the double Atwood machine can be understood as a serial connection of equivalent masses (see Example 1, Figure 4 in Matehkolaee, 2011). The series and parallel connections of Atwood machines can be studied by applying the CPE since, in the absence of dissipative forces, only transformations of gravitational potential energy and kinetic energy occur.

Figure 1

An Atwood machine composed of n *identical masses* m *connected in series, together with an additional mass* x *attached to the final pulley*

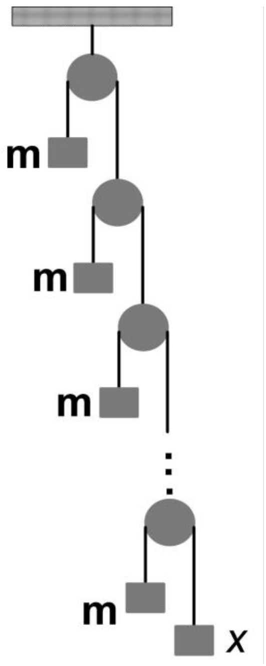

Figure 1 shows an Atwood machine conformed by n equal masses of mass m placed in series together with a mass x in the last pulley. The equivalent mass for this system can be expressed as Eq. (15) (Matehkolaee, 2011):

$$\left(f \circ f \underbrace{\cdots}_{order\ n} f\right)(x) = \frac{4^n x}{1 + \frac{(4^n - 1)x}{3m}}. \tag{15}$$

Substituting the specific values $n = 1, 2, 3, 4$ into (15) and considering the case $x = m$, the following results are obtained for the function f(x):

- For $n = 1$, it follows that $f(m) = 2m.$
- For $n = 2$, it follows that $f(m) = \frac{8}{3}m.$
- For $n = 3$, it follows that $f(m) = \frac{32}{11}m.$
- For $n = 4$, it follows that $f(m) = \frac{128}{43}m.$

This series can be represented by the expression $f(m) = a_n m$, where a_n is given by Equation (16):

$$a_n = \frac{3 \times 2^{2n-1}}{1 + 2^{2n-1}} \tag{16}$$

The general expression derived herein for a system of $n+1$ identical masses suspended from n pulleys in series (16) constitutes an original development. The closed-form expression for a_n provides a compact and scalable representation of the equivalent mass as a function of the number of masses, extending the treatment beyond specific low-order cases.

Pedagogically speaking, this result reinforces the central idea of the article since it shows that the concept of series connection, commonly associated with the physical systems shown in Table 1, can be meaningfully transferred to other systems through conservation principles. This could help students recognize cross-domain invariants and strengthen conceptual integration rather than isolated procedural learning.

Can two devices in series or in parallel yield the same equivalent physical quantity?

A relevant question addressed in this work is the following: *Is it possible to find any pair of devices whose equivalent physical quantity has exactly the same value when they are connected in series as when they are connected in parallel?*

This question is important because, in these cases, the expression for the equivalent physical quantity (*e.g.*, *Req*, *Ceq*…) always reduces to a simple mathematical form, either a linear function or a hyperbolic function. To address the question, the case of two resistors connected in series and in parallel is examined. It is well known that the equivalent resistance R_{eq} of two resistors in series and in parallel is given by Equations (17) and (18):

$$R_{eq} = R_1 + R_2 \tag{17}$$

and

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}, \tag{18}$$

respectively. Redefining $R_1 = x$ and $R_2 = y$, the last two equations take the following form:

$$y = R_{eq} - x, \tag{19}$$

for the series case, and

$$y = \frac{R_{eq} \cdot x}{x - R_{eq}} \tag{20}$$

for the parallel case. While keeping the value of R_{eq} constant, Equation (19) represents a linear function, whereas Equation (20) corresponds to a hyperbola, considering y as a function of x. Therefore, it is important to note that different physical connections generate distinct mathematical structures. This is not merely an algebraic curiosity; it reflects the fundamentally different ways in which resistors combine.

According to Joule's law, the dissipated power, $P = V^2/R$, is always positive, which implies that $R > 0,$ meaning that the device absorbs energy. This establishes a conceptual restriction on the domain of Equations (19) and (20): $0 < x < R_{eq}$.

Figure 2 shows both functions for a fixed value $R_{\mathrm{eq}} = 10\ \Omega$. The graphs do not intersect within the physically meaningful domain. Moreover, attempting to force an intersection would require a negative resistance in the parallel case, which is physically inadmissible in ordinary passive circuits. Thus, it is concluded that, for a given R_1, it is not possible to find an R_2 that, when combined either in series or in parallel, yields the same R_{eq} .

Figure 2

Resistance 2 (≡ y) as a function of resistance 1 (≡ x) with R_{eq} = 10 Ω

R_{eq}The same conclusion can be alternatively reached by assuming, hypothetically, that both connection types yield the same equivalent resistance R_{eq}. Thus, equating (19) and (20) yields

$$x + y = \frac{xy}{x+y};$$

which, after rearranging, produces

$$x^2 + y^2 + xy = 0.$$

The expression above is a sum of strictly positive terms (since $x > 0$ and $y > 0$). Therefore, it cannot vanish in the real numbers, *i.e.*, this expression has no physical solution in the real numbers. The impossibility is not merely algebraic; it stems from the positivity of physical resistance.

In addition, series and parallel connections obey intrinsic inequalities: for positive resistances, the equivalent resistance in series is greater than each individual resistance ($R_{serie} > \max(R_1,R_2)$), whereas, in parallel, it is smaller than either of them ($R_{paralelo} < \min(R_1,R_2)$). These relations follow from the physical distribution of current and potential, and they preclude a common positive value of R_{eq} for both configurations. Hence, although the equation $x^2 + y^2 + xy = 0$ emerges algebraically, it lies outside the physically admissible domain $x > 0$, $y > 0$.

It is worth emphasizing that the discussion in this subsection connects the algebraic structure, the graphical representation, and the physical meaning within a coherent conceptual framework.

Finally, in this study, we present a didactic proposal framed within a constructivist approach, aimed at promoting meaningful learning about how conservation laws operate in the analysis of electric circuits arranged in series and in parallel. The proposal, designed for undergraduate students in introductory and intermediate physics courses, integrates the POSSE strategy to activate prior knowledge and support comprehension (Diquito *et al.*, 2025). This strategy is combined with the use of interactive computer simulations from the PhET environment (University of Colorado Boulder, 2025), which have been shown to be effective tools for fostering conceptual understanding in physics within a constructivist framework. The overall structure of the instructional sequence is described in detail in the supplementary material.

CONCLUDING REMARKS

The 15 series and parallel connections analyzed in this study (including those presented in Table 1 and the one corresponding to the Atwood machine) can be studied in an integrated and unified manner, using four conservation principles: energy, mass, charge, and linear momentum. This approach contributes to students developing what Gardner (2008) calls a *disciplined mind*, as they recognize an important physics concept that enhances physical thinking and promotes meaningful learning. Students can associate new knowledge with previous understanding rather than being limited to fragmented or mechanical learning.

The series or parallel connections of Atwood machines can be studied based on the CPE by incorporating the concept of *equivalent mass*. In this work, a general expression was obtained for the equivalent mass when $n + 1$ equal masses are suspended from *n* pulleys in series.

Furthermore, we examined the series and parallel connections of isolated device pairs, given that, in these cases, the equivalent physical quantity expressions always take the form of either a linear function or a hyperbola. It was demonstrated that, for a given R_1, there is no R_2 that can be combined in series or in parallel to produce the same equivalent resistance R_{eq}.

The primary limitation of this study is the absence of experimental validations that document the impact of the proposed approach on student learning. Nevertheless, the main goal of this manuscript is to formulate a theoretically grounded instructional proposal for physics education, articulating how conservation laws can function as a unifying thread that connects concrete systems with more abstract formal contexts. This proposal is contained in the supplementary material.

Future research could extend this work to empirical studies in real classroom settings in order to evaluate the effectiveness of the proposed instructional strategy. In addition, other potential articulating topics in physics could be identified which promote conceptual coherence and reduce fragmentation in learning.

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