



## Research

### Scheduling in a Simple Assembly Robotics Cell to Minimize Earliness and Tardiness

#### Programación de trabajos en una celda robótica de ensamble simple para minimizar los adelantos y las tardanzas

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#### Abstract

**Background:** Robotics Assembly Cells (RAC) have been designed to meet the flexibility requirements demanded by today's globalized market. The objective is to manufacture a vast variety of products at a low cost, which requires equipment with a high level of flexibility, such as robots. The need to schedule a great variety of jobs in an RAC is a very relevant issue, as efficiency and productivity depend on the sequence in which jobs are scheduled. Studies around this matter have developed models with analytical and heuristic approaches, as well as simulation methods and genetic algorithms, seeking to improve performance measures based mainly on time, utilization, and costs.

**Method:** The purpose of this article is to formulate an exact mathematical model using mixed-integer linear programming (MILP) to optimize small scheduling problems. The objective is to minimize the measure of performance related to the tardiness and earliness of jobs. This optimization aims to mitigate the effects of delays in product deliveries, queue times, and work-in-process inventory in subsequent processes. Doing so facilitates adherence to agreed-upon delivery deadlines and prevents bottlenecks in the assembly cell.

**Results:** The proposed mathematical model generates optimal solutions to the job scheduling problem in the assembly cell, which serves as a case study. This addresses the need to minimize tardiness to meet delivery deadlines or minimize earliness while avoiding an increase in work-in-process inventories. The model ensures that optimal scheduling decisions are made to optimize both delivery performance and inventory levels.

**Conclusions:** Due to the NP-hard complexity of the scheduling problem under study, the proposed mathematical model demonstrates computational efficiency in solving scheduling problems with fewer than 20 jobs. The model is designed to handle such smaller-scale problems within a reasonable computational time frame, considering the inherent complexity of the scheduling problem.

**Keywords:** robotic assembly cell, tardiness, scheduling

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## Resumen

**Contexto:** Las celdas robóticas de ensamble RAC han sido diseñadas para cumplir con los requerimientos de flexibilidad que exige el mercado globalizado actual. El objetivo es fabricar una alta variedad de productos a bajos costos, por lo cual se requiere de equipos con un alto nivel de flexibilidad como los robots. La necesidad de programar una amplia variedad de trabajos en una RAC es un problema muy relevante, dado que la eficiencia y la productividad dependen de la secuencia en la cual se programan los trabajos. Los estudios alrededor de este asunto han desarrollado modelos con enfoques analíticos y heurísticos, así como métodos de simulación y algoritmos genéticos, que buscan mejorar medidas de desempeño basadas principalmente en el tiempo, el grado de utilización y el costo.

**Método:** El propósito de este artículo es formular un modelo matemático exacto mediante programación lineal entera mixta (MILP) para optimizar problemas pequeños de programación. El objetivo es minimizar las medidas de desempeño de tardanza y adelanto de los trabajos. Esta optimización busca mitigar los efectos de las demoras en las entregas de los productos, los tiempos de cola y los inventarios de producto en proceso en instancias posteriores. Esto permite cumplir con los plazos de entrega pactados y evita bloqueos en la celda de ensamble.

**Resultados:** El modelo matemático propuesto genera soluciones óptimas al problema de programación de trabajos en la celda de ensamble propuesta, lo que sirve como caso de estudio. Esto aborda la necesidad de minimizar las tardanzas para cumplir con los plazos de entrega o de minimizar los adelantos y evitar un aumento en los inventarios de producto en proceso. El modelo garantiza que se tomen decisiones óptimas de programación tanto para mejorar los tiempos de entrega como los niveles de inventario.

**Conclusiones:** Debido a la complejidad NP-difícil del problema de programación estudiado, el modelo matemático propuesto demuestra eficiencia computacional en la resolución de problemas de programación con menos de 20 trabajos. El modelo está diseñado para manejar tales problemas de menor escala dentro de un marco de tiempo computacional razonable, considerando la complejidad inherente del problema de programación.

**Palabras clave:** celdas de ensamble robóticas, tardanza, programación.

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## 1. Introduction

The current market demands increasingly personalized products at mass production prices. This poses a significant challenge for industries worldwide. To remain competitive in a globalized market, industries have implemented flexible manufacturing systems (FMS). FMS allows industries to meet market demands and reduce manufacturing costs, ultimately leading to profitability. FMS offer several advantages, particularly in terms of flexibility, which refers to a system's ability to manufacture a wide range of products using the same equipment, with short setup times (1). FMS are generally classified into two main subgroups: flexible assembly systems (FAS) and flexible manufacturing systems (FMS) (2). Much of the research conducted in the field has predominantly focused on FMS, while FAS have received comparatively less attention. Both FAS and FMS are categorized as computer-integrated manufacturing systems, but they exhibit distinct differences in several aspects. Firstly, FAS are capable of processing a significantly greater number of diverse tasks compared to FMS. In FAS, multiple components and parts are joined simultaneously, whereas FMS typically operate on one part at a time. Secondly, the processing times required for each operation in FAS are shorter than in FMS. Consequently, the relationship between transfer and processing times in FAS are generally higher in comparison with that of FMS. Thirdly, the material handling systems of FAS tend to be more complex. These differences highlight the unique characteristics and operational variance observed between FAS and FMS, suggesting the need for dedicated research and analysis to understand and optimize the performance of both systems (3). Table I shows some differences between FAS and FMS, which make issues in the former more difficult (4).

**Table I.** Differences between FAS and FMS

Features	FAS	FMS
Number of different tasks that can be performed	High	Low
Number of pieces per job	Several	Only one
Processing time per part	Short (seconds)	Long (minutes/hours)
Setup time/processing time ratio	High	Low
Material handling	Complex (assembly operations)	Simple (loading/unloading)
Collaborative work man/machine	High	Low

FAS can be classified into two types: robotic assembly lines (RAL) and robotic assembly cells (RAC) (5). A RAL is a line flow system that comprises assembly stations interconnected in series through an automated material handling system. It is primarily employed for assembling products characterized by a high volume and a low variety. These products typically have stable designs and exhibit minimal fluctuations in demand requirements (6). The RAC is a highly advanced system that incorporates robots, assembly stations, and an automated material handling system, all under computerized control. RACs are designed to efficiently assemble a wide range of products in small batches. One of the key advantages of an RAC is its ability to utilize one or more robots that can perform multiple operations simultaneously or collaboratively. This enables greater flexibility and productivity within the cell (7). Fig. 1 shows the layout of a RAL and a RAC.

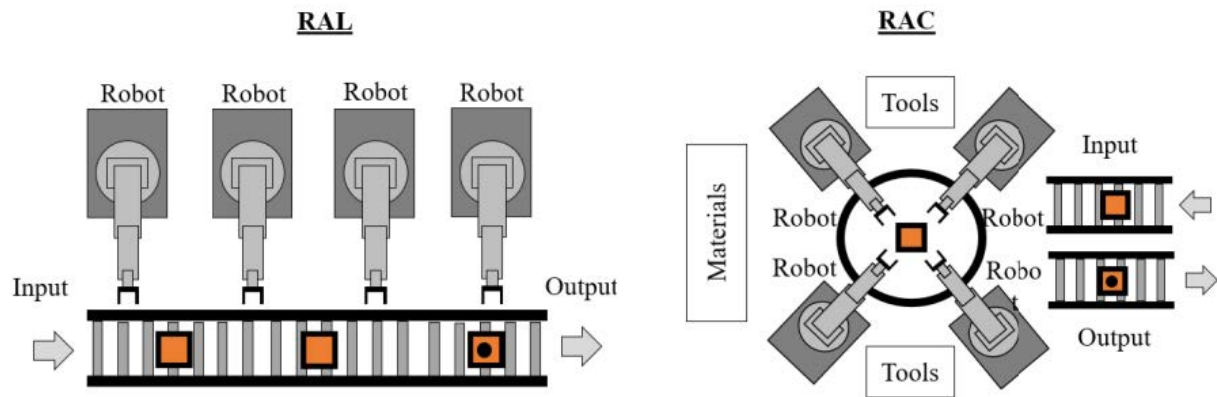


Figure 1. Layout of a RAL and a RAC

Scheduling is a decision-making process that plays a vital role in most manufacturing industries. Its function is to optimize the allocation of limited resources for job processing. These resources include machines, robots, tools, material handling equipment, and materials. A job consists of a series of operations or tasks to be performed by manufacturing systems (8). Few studies have addressed the issue of task scheduling in RACs. These studies can be classified into three approaches: the analytical and heuristic approach, the simulation approach, and genetic algorithms. In addition, studies have used different types of performance measures to evaluate the results of scheduling. Measures based on time, utilization, and costs are the most widely used, and time-based performance measures that has attracted the most attention (9). Most of real-life scheduling issues are difficult to solve with a non-deterministic polynomial time NP-hard model and tend to require a large amount of constraints to generate a reliable solution (10).

Scheduling issues in manufacturing systems are specified by a set of elements. The critical elements are decision variables, constraints, and objective functions. The goal of any manufacturing company is to maximize the utilization of resources, minimize completion times, and meet the due dates of orders (11). In field of scheduling, several time-based objective functions are used to evaluate the performance of a system under different scheduling strategies, such as makespan, flow time, tardiness, idle time, queue time, waiting times, and setup times, among others (12). One fundamental aspect when it comes to managing an RAC is to meet the product's due date, either to deliver the final product to the customer on time or to deliver the semi-finished product to the next process on time, thereby not delaying the system. This job becomes complex when, in a RAC, multiple products with different due dates must be assembled. Here, the problem lies in being able to determine the processing sequence of the tasks that help minimize potential tardiness. Tardiness is defined as the amount of time by which a job exceeds its due date. In scheduling theory, three measures of tardiness are estimated: total tardiness ( $\sum T_j$ ), the sum of the tardiness of all late jobs; maximum tardiness ( $T_{max}$ ), the value of the job that obtained the longest tardiness; and the number of tardiness jobs (NT) the amount of jobs that were delivered late (13). The other criterion is earliness, defined as the amount of time by which a job is ahead of its due date. This is a very important and frequent industrial problem that is common to most just-in-time (JIT) production environments. JIT consists of delivering products and

services at the right time for immediate use, where the main objective is the continuous search for improving the production process. This is achieved and developed through reduced inventories. JIT scheduling problems are very common in the industry. In a JIT scheduling environment, a job should be finished as close to the due date as possible. An early job completion results in inventory carrying costs, such as those related to storage and insurance (14). Balancing both tardiness and earliness is essential in achieving overall optimization in a RAC. The scheduling algorithm aims to strike a balance between completing jobs within their due dates to avoid tardiness and minimizing excessive earliness to optimize resource utilization. By considering both tardiness and earliness as criteria, manufacturers can effectively manage the assembly process in RACs. Striking the right balance ensures that jobs are completed within the desired time frames, optimizing production efficiency and meeting customer demands without unnecessary delays or excess inventory. Scheduling issues are theoretically classified according to various criteria, such as production volume, the nature of production, production capacity, and manufacturing systems. Each type of scheduling issue has different levels (15, 16).

It has been previously mentioned that one of the challenges in scheduling tasks for a RAC is meeting the due dates for an assigned job. In the scheduling area, the tardiness  $T_j$  is calculated as the maximum value of the subtraction between the completion time of the job  $C_j$  minus the due date  $D_j$  (8). This is calculated via Eq. 1:

$$T_j = \max(C_j - D_j; 0) \quad (1)$$

If job  $j$  is finished before the due date, the tardiness is 0, since none was generated. Tardiness does not take negative values; if that were to happen, we would be dealing with earliness. On the other hand, the earliness  $E_j$  is calculated as the maximum value of the subtraction between the due date  $D_j$  minus the completion time of the job  $C_j$  (8). This is calculated via Eq. 2:

$$E_j = \max(D_j - C_j; 0) \quad (2)$$

It is worth remembering that three measures of earliness are estimated: the total earliness ( $\sum E_j$ ), which is the sum of the earliness of all early jobs; the maximum earliness ( $E_{max}$ ), which is the value of the job that obtained the maximum earliness; and the number of earliness jobs ( $NE$ ), which is the amount of jobs that were delivered early.

Scheduling issues for evaluating tardiness and earliness have been solved using heuristic algorithms, exact models, and expert systems (13). The most significant theoretical developments based on heuristic algorithms for the  $1||T_j$  problem are the Emmons dominance conditions, known as Emmons theorems, which determine precedence relationships between jobs to generate an optimal sequence. The first Emmons theorem provides the necessary conditions for a shorter job to precede a longer one in an optimal sequence, while the second theorem provides the necessary conditions for a longer job to precede a shorter one in an optimal sequence. These relationships have been extensively used to reduce the solution space in enumeration methods (17). Lawler's decomposition theorem is the second most significant algorithm. In order to explain the decomposition theorem, let us suppose that the jobs are numbered in the order of the earliest due date or (EDD). According to Lawler (18), the  $1||T_j$  problem is decomposed with the longest  $j$  job in a  $k$  position. This author makes the following assumption: the longest  $j$  job is completed as tardy as possible in an optimal sequence, which means moving the jobs

from left to right while meeting some conditions. The development of these algorithms gave way to several studies, where dispatch rules were defined based on the execution and delivery times of jobs. The most used dispatch rules to solve the aforementioned problem are:

- *First come, first served (FCFS): jobs that arrive first are scheduled first.*
- *Earliest Due Date First (EDD): jobs with the earliest due date are scheduled first.*
- *Shortest Processing Time (SPT): jobs with the shortest processing time are scheduled first.*
- *Longest Processing Time (LPT): jobs with the longest processing time are scheduled first.*
- *Slack Time Remaining First (STR): jobs with the minimum slack time remaining are scheduled first. As shown in Eq. 3:*

$$STR = D_j - P_j \quad (3)$$

where:

$D_j$ : due date of job  $j$

$P_j$ : processing time of job  $j$

There are many examples where the penalty for a tardiness job remains the same, no matter how long the tardiness is. In this case, the objective is to minimize the number of tardy jobs  $NT$ . Moore's algorithm (1968) minimizes the number of tardy jobs for the problem of a single machine, where all jobs have a same release time. This algorithm consists of four very simple steps. In the first step, the jobs are sequenced while following the EDD rule. Then, the tardiness jobs are ordered according to the LPT rule until an optimal solution is reached (19). Most of the exact algorithms for the  $1||T_j$  problem use the dynamic programming (DP), the branch and bound method (B&B), or a hybrid DP/B&B approach. The most efficient of these algorithms is the Potts and Van Wassenhove (P-W) algorithm, capable of solving problems with up to 100 jobs. The P-W algorithm decomposes the problem until the generated subproblems are small enough to be solved via DP, and it does not use any lower bound (20). The best exact algorithms to solve  $1||T_j$  are the B&B, one that use the latest developments of the Emmons and Lawler decomposition theorems. The performance of these algorithms can be improved by using induced due dates, calculated after certain jobs have been shown to precede/follow others. Szwarc's branching algorithm (2001) is an example of such algorithm, and it can handle problems with up to 500 jobs. In addition, in the last decade, several algorithms based on metaheuristics and artificial intelligence have been developed to solve tardiness minimization problems, considering more variables and constraints for tasks and machines. These algorithms represent a complex computational development (13).

In the literature, earliness and tardiness penalties are studied by various authors from a single-objective point of view. Most works consider distinct or common due dates.

(21) studied the problem considering distinct due dates. They presented an optimal algorithm with polynomial complexity to determine the optimal completion time for each job in a schedule determined by a genetic algorithm (GA). This optimal algorithm was used because it may be interesting to anticipate a job, even paying a penalty if it is shorter than that generated by the tardiness.



(22) presented a hybrid genetic-bees algorithm (GBA)-optimized solution for minimizing the weighted sum of earliness and tardiness penalties in the context of the single machine scheduling problem (SMSP). The bee algorithm (BA) was improved by incorporating GA operators during the global search stage, aiming to progressively enhance the BA's global search capability with new additions.

(23) addressed the SMSP while aiming to formulate schedules for tasks with arbitrary due dates and minimizing the total earliness/tardiness with respect to said due dates. The problem was approached through three distinct formulations: 1) where the start time of the machine was fixed, 2) where the start time belonged to a specified time segment, and 3) where the start time was arbitrary. Given the inherent complexity, there is no exact polynomial algorithm for its solution. For the first two formulations, the PSC-algorithms were presented. Each algorithm incorporates clear indicators of feasible solution optimality and is grounded upon optimal solutions for the single machine problem, specifically designed to minimize the total tardiness of tasks relative to their varied due dates with equal weights.

(24) addressed the SMSP to minimize the total weighted earliness and tardiness relative to a nonrestrictive common due date. This problem is fundamental and has applications in JIT manufacturing. It is connected to a boolean programming problem with a quadratic objective function, referred to as the half-product. This research presented an approach to create a fast fully polynomial-time approximation scheme (FPTAS) for this problem, and the running time was aligned with the best-known running time for an FPTAS aimed at minimizing a half-product with no additive constant.

(25) presented a mixed-integer programming model for a SMSP whose objective was to minimize the total earliness/tardiness duration when the uncertainty of parameters such as processing times and due date was coded with grey numbers. Grey theory and grey numbers serve as tools for representing the uncertainty associated with parameters such as processing times and common due dates. The authors proposed a 0-1 mathematical model to address the problem, coupled with an efficient heuristic method that leveraged the expected processing times for job ordering.

(26) studied the SMSP while considering past-sequence-dependent setup times. This study explored scenarios involving common due date assignments, slack due dated, and different due date assignments. The objective function aimed to minimize the linear weighted sum of earliness-tardiness, the count of early and delayed jobs, and due date costs. The optimal properties of the problem were outlined, and it was proven that the problem is solvable in polynomial time. Additionally, three extensions to the problem were proposed: incorporating assumptions of position-dependent, time-dependent, and position-and-time-dependent processing times.

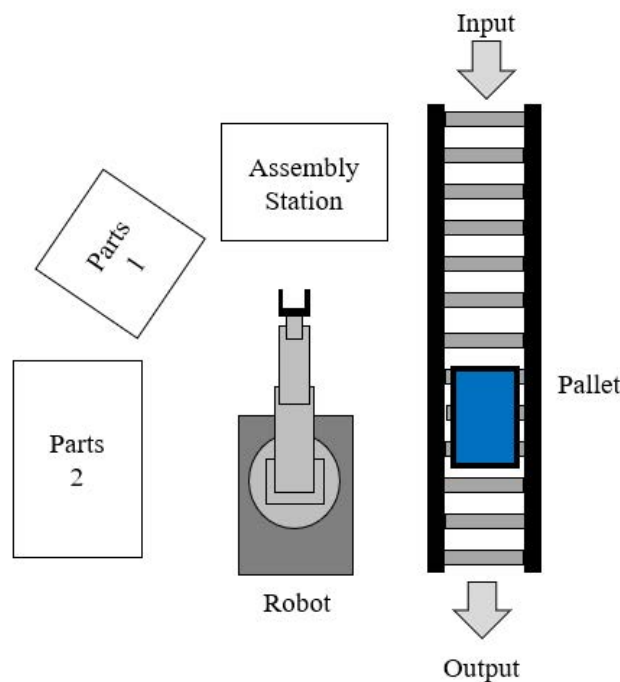
The objective of this article is to develop a MILP model that aids in scheduling and sequencing jobs within a RAC. The aim is to minimize both tardiness and earliness penalties, considering the presence of various job release times. By formulating this new MILP model, this article seeks to generate

an optimal solution that surpasses the quality of the results obtained through heuristic approaches and alternative algorithms like the B&B method. This approach offers the advantage of reduced computational complexity. The inclusion of different job release times adds a level of complexity to the problem, which the MILP model aims to effectively address.

## 2. Methodology

This study was conducted at the flexible manufacturing laboratory of Facultad de Ciencias Empresariales, Universidad Tecnológica de Pereira. Here there is a RAC called simple assembly robotized cell (SRAC) because it only has one robot. It was designed with the ability to assemble the same family of products, so the robot does not require a gripper change. There are no transfer or setup times, nor trajectory collisions analyzed since the cell is made up of a single robot. Thus, the number of variables is reduced, making it possible to simplify the mathematical model to perform the first tests.

The SRAC is composed of one robot, which takes the components to be assembled from two deposits called Parts 1 and Parts 2. The combination of different parts allows assembling up to 12 different products without the need for reconfiguring of the robot. The task of joining different parts is carried out in the assembly station, and then, when the process complete, the product is positioned on the pallet to be transported to the next stages of the process. SRAC is the first stage of a three-stage process to complete a finished product. Fig. 2 shows a description of the SRAC.



**Figure 2.** Simple assembly robotic cell (SRAC)



For the first test, the scheduling of 6 jobs was taken as a reference. One job involved a standardized lot of the same product the size of each lot was previously determined to minimize transfer times between jobs. Thus, each time a product needed assembly, all the lot was assembled. Since the SRAC is the first stage in the process, and assuming that the Parts 1 and Parts 2 deposits are always loaded with the required parts, the assembly process does not start until the part from the raw material warehouse arrives at the workstation, which causes job release times to vary. The due dates  $D_j$  for the jobs to be assembled at the SRAC were determined through an MRP program that determines due dates according to production orders generated by customer orders. Table II shows the requirements of the jobs to be scheduled in the first run:

**Table II.** SRAC requirements

Cycle time [minutes/unit]	Jobs					
	1	2	3	4	5	6
Cycle time [minutes/unit]	10	6	8	8	5	3
Lot size [units]	10	5	5	10	20	20
Processing time $P_j$ [minutes]	100	30	40	80	100	60
Due date $D_j$ [minutes]	150	60	95	230	200	300

Once the information on production requirements was obtained, job scheduling was carried out while following the four dispatch rules described above (FCFS, EDD, SPT, LPT and STR). For the sequencing of the 6 assembly jobs to be processed by the SRAC, there were a total of  $6!$  possible combinations, *i.e.*, 720 different sequencing possibilities. The goal was to find the sequence of jobs that minimizes both earliness and tardiness. An exact mathematical model was formulated, capable of solving the job sequencing problem, with the objective of minimizing the earliness ( $\alpha$ ) and tardiness ( $\beta$ ) penalties. The results obtained with the model were evaluated, validated, and compared against the performance obtained through five dispatch rules.

### 3. Development

To formulate the MILP problem, the value of the  $\alpha$  and  $\beta$  penalties was taken, which depends on the problem to be addressed. If minimizing the work-in-process (WIP) is required after assembly at the workstation, the value of  $\alpha$  must be greater than  $\beta$ . On the contrary, if one wishes to minimize job delays to shorten the makespan of the process, then  $\beta$  must be greater than  $\alpha$ . To generate the MILP model, the following variables were defined:

$j$  : subscript that identifies the job to be scheduled

$i$  : subscript that identifies the position in the schedule

$\alpha_j$  : earliness penalties for job  $j$

$\beta_j$  : tardiness penalties for job  $j$

$r_j$  : release time for job  $j$

$X_{ij}$  : binary variable, 1 if job  $j$  is performed at position  $i$ , 0 otherwise

- $P_j$  : processing time for job  $j$   
 $C_i$  : completion time for job scheduled at position  $i$   
 $D_j$  : due date for job  $j$   
 $T_j$  : tardiness for job  $j$ , no negativity  
 $E_j$  : earliness for job  $j$ , no negativity  
 $Y_j$  : binary variable, 1 if job  $j$  has tardiness, 0 else  
 $Z_j$  : binary variable, 1 if job  $j$  has earliness, 0 else

Once the job schedules had been obtained by applying heuristic dispatch rules, the following question arose: Is it possible to develop a mathematical model which allows finding a sequence of jobs that minimize the total for earliness and tardiness penalties? The model proposed to answer this question is presented below.

Objective function:

$$\text{Min} = \sum_{j=1}^N \alpha_j \cdot E_j + \sum_{j=1}^N \beta_j \cdot T_j \quad (4)$$

subject to:

$$\sum_{j=1}^N (P_j \cdot X_{ji}) + C_{i-1} \leq C_i \quad \text{for all } j \quad (5)$$

$$\sum_{i=1}^N (C_i \cdot X_{ji}) - D_j = T_j \cdot Y_j - E_j \cdot Z_j \quad \text{for all } j \quad (6)$$

$$\sum_{i=1}^N (C_i \cdot X_{ji}) \geq r_j + P_j \quad \text{for all } j \quad (7)$$

$$\sum_{i=1}^N X_{ji} = 1 \quad \text{for all } j \quad (8)$$

$$\sum_{j=1}^N X_{ji} = 1 \quad \text{for all } i \quad (9)$$

$$X_{ji}; Y_j; Z_j : \text{Binary};$$

$$T_j \geq 0; E_j \geq 0;$$

Eq. 4 shows the definition of the objective function. Eq. 5 calculates the completion time of the scheduled job at position  $i$ . Eq. 6 calculates the earliness or tardiness of job  $j$ . If the earliness is greater than zero, the binary variable  $Z_j$  takes a value of 1, and, if the tardiness is greater than 0, the binary variable  $Y_j$  takes a value of 1. Eq. 7 establishes that the completion time of job  $j$  is greater than the release time plus the processing time when there are different job release times. Eq. 8 determines that a job  $j$  can only be scheduled in a single position  $i$ . Eq. 10 states that, in a position  $I$ , only one job  $j$  can be scheduled.

In some cases, the penalty for having an early or tardy job is independent of the time of earliness or tardiness; only having an early or tardy job is penalized. In this case, the objective function is formulated as follows:

Objective function:

$$\text{Min} = \sum_{j=1}^N \alpha_j \cdot Z_j + \sum_{j=1}^N \beta_j \cdot Y_j \quad (10)$$

## 4. Evaluation

To evaluate the proposed MILP model, the performance of the earliness and tardiness penalties was evaluated in relation to the dispatch rules (FCFS, EDD, SPT, LPT, and STR). This section lists, the most important assumptions made in this study:

- The workstation can perform only one job at a time.
- Process pre-emption is not allowed.
- The required sequence of machines and the processing times of the jobs are known.
- There is no restriction regarding queue length at the workstation.
- There are no interruptions at the workstation and no machine breakdowns.
- There are no setup times for jobs.
- There are no alternate routings.
- There are no limiting resources other than the workstation.

To evaluate the proposed model, we executed one hundred instances, divided into three stages. Here is a summary of the stages and the parameters used.

- **Stage one.** Constant penalty values were assumed to analyze the system's performance when the penalty for tardiness is greater than that for earliness and vice versa. The penalty values used were  $\alpha = 5$  and  $\beta = 10$ . In the second instance, the penalty values used were  $\alpha = 10$  and  $\beta = 5$ .
- **Stage two.** Different penalty values for each job, denoted as  $\alpha_j$  and  $\beta_j$ , were evaluated. These penalty values were generated by drawing data from a uniform distribution with a range of [5-20]. This allowed for a more varied and dynamic evaluation of the system.
- **Stage three:** The objective function was evaluated to minimize the number of tardy and early jobs, considering the penalty values  $\alpha_j$  and  $\beta_j$  generated by the same uniform distribution system in stage two.

The instances of this stage were divided into five groups based on the number of jobs: 6, 10, 15, 18, and 20. For each stage, several parameters were considered to create the instances:

- **Job arrivals.** The arrivals of jobs were generated using an exponential distribution.
- **Processing times.** The processing times for each job were drawn from a rectangular distribution within the range of [20-100].

- **Due date.** The due date of each job was determined via an assignment factor ( $k$ ), which was randomly selected from a uniform distribution within the range of [2-10]. The due date calculation equation was used to determine the specific due date for each job, as shown below.

$$D_j = (r_j + P_j)k \quad (11)$$

By varying the penalty values, the number of jobs and the distribution of job arrivals, process times, and due dates, this study aimed to comprehensively evaluate the proposed model's performance under different scenarios and conditions. The results obtained for the first 50 instances with constant values of  $\alpha$  and  $\beta$  are shown in Table III.

**Table III.** Evaluation with  $\alpha = 5; \beta = 10$  and  $\alpha = 10; \beta = 5$

$\alpha = 5; \beta = 10$								$\alpha = 10; \beta = 5$							
Jobs	Run	FCFS	EDD	SPT	LPT	STR	MILP	Jobs	Run	FCFS	EDD	SPT	LPT	STR	MILP
6	1	3 075	1 500	2 125	6 100	2 325	1 400	6	1	3 075	1 500	2 225	3 950	1 725	1 150
	2	3 500	3 750	3 475	4 950	4 750	2 950		2	2 650	2 850	2 975	2 925	2 825	2 150
	3	2 275	2 250	1 625	6 300	1 525	1 525		3	2 150	2 025	1 900	4 125	1 700	1 300
	4	3 425	1 825	2 750	5 050	1 625	1 625		4	3 325	1 775	3 100	3 650	1 375	1 175
	5	4 375	2 775	3 575	5 500	3 425	2 575		5	3500	2 100	3 100	3 500	2 125	1 825
10	6	14 000	7 350	7 250	16 880	6 395	6 095	10	6	8 650	3 900	4 300	9 880	3 445	3 295
	7	10 125	10 450	9 250	18 400	9 475	7 500		7	5 475	5 600	5 000	10 550	5 225	4 375
	8	5 220	3 070	4 620	19 145	3 290	3 070		8	6 425	4 800	6 075	9 150	5 125	3 675
	9	6 525	3 725	4 050	12 550	3 525	3 225		9	5 175	3 350	4 650	9 425	3 350	3 300
	10	10 325	5 300	5 825	16 750	5 500	5 100		10	6 325	3 250	3 925	9 525	3 350	3 100
15	11	16 100	5 675	12 050	16 950	8 500	5 150	15	11	15 300	5 825	7 900	16 700	6 700	5 525
	12	19 775	10 675	12 050	26 950	12 500	6 275		12	18 550	5 050	10 200	21 850	6 700	2 175
	13	14 900	3 600	10 975	24 600	4 050	3 300		13	14 725	3 675	12 500	18 075	3 525	2 875
	14	16 500	5 050	10 725	24 200	5 750	2 875		14	12 200	6 075	10 250	14 500	8 050	3 025
	15	12 775	8 050	10 500	18 750	6 525	3 225		15	16 575	5 025	14 225	16 575	6 050	3 525
18	16	16 375	4 225	9 125	18 275	4 225	2 350	18	16	17 400	8 450	14 350	18 650	8 450	4 375
	17	15 950	6 650	11 750	24 475	8 150	4 750		17	16 450	6 700	13 600	20 625	5 650	3 350
	18	18 450	8 050	12 525	20 750	7 525	5 225		18	18 550	7 050	12 775	20 250	8 220	3 575
	19	19 550	6 325	14 075	22 775	6 850	3 825		19	16 750	8 075	14 225	18 450	9 025	5 050
	20	16 450	6 775	15 850	22 150	6 785	4 325		20	18 025	7 520	12 250	20 550	8 050	3 775
20	21	32 300	21 175	29 700	38 525	22 650	14125	20	21	42 850	18 770	28 075	52 620	22 650	6 725
	22	34 820	18 075	28 250	42 250	16 780	10 530		22	47 620	2875	32 425	50 850	26 530	8 860
	23	36 875	20 450	26 750	41 880	22 720	12 375		23	40 885	22 455	34 050	48 355	22 860	10 525
	24	38 200	22 560	34 225	44 530	24 350	10 850		24	46 200	28 360	32 460	52 160	21 225	8 735
	25	36 350	22 050	28 450	42 225	21 335	12 725		25	48 750	24 550	32 285	54 730	25 925	10 220

In this first stage of evaluation, it can be observed that dispatch rules based on delivery deadlines, such as EDD and STR, perform best with responses close to the optimum obtained with the MILP model. These two dispatch rules aim to minimize tardiness, so their performance is better when the value of  $\alpha$  is lower than the value of  $\beta$ . When  $\alpha$  is greater than  $\beta$ , it can be noted that the gap between the two rules and the optimum value increases. Figs. 3 and 4 show the comparative curves for the first stage of evaluation.

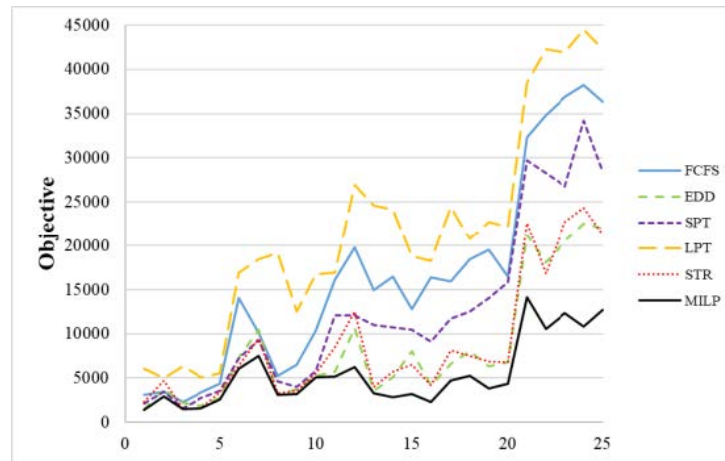


Figure 3. Performance with  $\alpha = 5$  and  $\beta = 10$

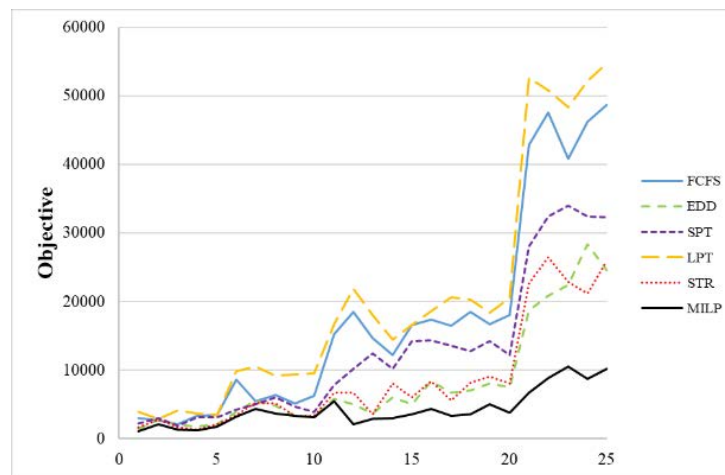


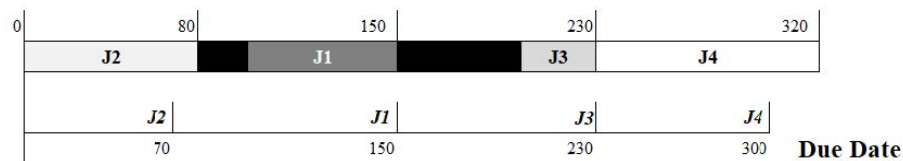
Figure 4. Performance with  $\alpha = 10$  and  $\beta = 5$

For the second stage of evaluation, random values of  $\alpha$  and  $\beta$  were estimated for each job, as described above. The results obtained are shown in Table IV.

The evaluation shows that when the penalty values  $\alpha$  and  $\beta$  are variable and dependent on each job  $j$ , the gap between the optimal performance and the analyzed dispatch rules is much larger compared to when constant penalty values  $\alpha$  and  $\beta$  were evaluated. This is because the model is able to prioritize jobs based on their penalty values. For example, a job with a high earliness penalty can be scheduled with some tardiness, or vice versa. Furthermore, the model is capable of scheduling jobs in such a way that both earliness and tardiness are equal to zero. This can be achieved by delaying the start of the job, which generates idle time in the assembly cell and allows minimizing the objective function. Fig. 5 provides an example of the described scenario. This analysis demonstrates the flexibility and effectiveness of the MILP model in handling variable penalty values and optimizing the scheduling of jobs to minimize both earliness and tardiness.

**Table IV.** Evaluation with  $\alpha_j$  and  $\beta_j$ 

$\alpha_j; \beta_j$							
Jobs	Run	FCFS	EDD	SPT	LPT	STR	MILP
6	1	2.335	2.885	3.245	4.535	3.340	1.855
	2	2.160	1.375	1.125	3.560	2.360	1.125
	3	5.275	1.575	1.750	5.775	1.775	1.525
	4	3.665	2.375	2.620	3.950	2.175	2.100
	5	3.025	2.825	2.850	6.350	2.825	1.775
10	6	10.590	4.135	3.930	15.825	4.230	2.410
	7	12.600	8.425	9.050	13.020	7.545	3.345
	8	11.375	8.080	11.400	23.825	8.080	4.675
	9	14.525	9.750	8.625	15.270	10.560	5.065
	10	10.285	6.230	6.890	11.475	6.230	3.225
15	11	25.355	7.075	16.260	39.275	6.375	2.825
	12	25.610	6.700	30.290	29.190	6.150	2.350
	13	40.845	16.460	19.345	39.825	20.365	6.440
	14	32.475	22.535	26.765	38.820	20.880	3.565
	15	18.200	12.500	19.400	35.600	13.500	1.825
18	16	38.350	19.225	24.625	44.750	18.125	5.175
	17	46.475	11.150	12.550	39.550	19.075	4.950
	18	38.600	14.350	17.750	45.925	14.150	2.275
	19	34.860	12.425	10.050	42.560	16.625	3.785
	20	32.245	16.600	22.875	38.630	17.775	4.025
20	21	52.765	24.765	32.665	60.225	26.860	6.865
	22	54.860	26.050	22.890	62.450	30.750	5.750
	23	58.930	22.175	28.330	66.750	22.815	4.270
	24	50.775	25.050	28.780	62.025	26.430	3.085
	25	58.450	20.885	20.250	64.340	22.315	6.315

**Figure 5.** Schedule with MILP model

The SPT rule exhibits a similar performance to EDD and STR, despite not considering delivery deadlines. The performance curves depicted in Fig. 6 provide a visual comparison between the dispatch rules and the MILP model, highlighting their relative performance in terms of minimizing tardiness and earliness penalties. In this figure, it can be observed that the SPT rule achieves a relatively close



performance to EDD and STR. SPT prioritizes jobs with shorter processing times, which can indirectly contribute to minimizing earliness and tardiness. While the SPT rule may not explicitly optimize tardiness or earliness, its performance is still competitive, showcasing its effectiveness as a simple and efficient dispatch rule under certain scenarios.

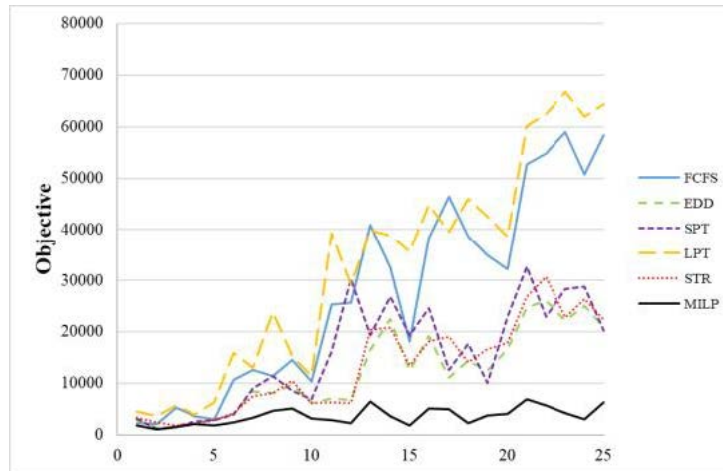


Figure 6. Performance with  $\alpha_j$  and  $\beta_j$

Finally, in the third stage, the MILP model was evaluated to minimize the penalty for getting a job early or late, regardless of the job's number of time units spent in tardiness or earliness. For this evaluation, the objective function described in Eq. 10 was taken as a reference. Table V shows the results of the assessment.

No significant differences can be observed between the dispatch rules used, and the gap with respect to the responses obtained by the MILP model increases as more jobs are scheduled. The evaluation results are presented in Fig. 7. It can be seen that, as the number of jobs increases, the performance of the dispatch rules becomes less competitive compared to the optimal solution obtained by the MILP model. This increasing gap highlights the computational superiority of the MILP model in handling larger scheduling problems and achieving more optimal results.

The MILP model demonstrated as significantly superior performance compared to the dispatch rules. However, it should be noted that the computation time of the model increases exponentially as the number of jobs increases. The model performs efficiently with a maximum of 20 jobs, but, beyond that, the computation time becomes impractical. Fig. 8 depicts the time curves obtained for the solution of the three types of analysis: constant  $\alpha$  and  $\beta$  values, variable  $\alpha_j$  and  $\beta_j$  values, and  $\alpha_j Z_j$  and  $\beta_j Y_j$  multiplied by the binary variables. The instances were run on a laptop with an Intel(R) Core (TM) i3-6100U CPU @ 2.30GHz, processor and 6 GB RAM. As expected, the computation time increases as the complexity of the analysis increases, especially when considering variable penalty values and the multiplication of binary variables.

These results highlight the computational limitations of the MILP model and emphasize the importance of considering scalability and computational resources when applying the model to larger

**Table V.** Evaluation with  $\alpha_j Z_j$  and  $\beta_j Y_j$ 

$\alpha_j Z_j; \beta_j Y_j$							
Jobs	Run	FCFS	EDD	SPT	LPT	STR	MILP
6	1	45	50	45	50	50	40
	2	40	50	75	40	50	35
	3	70	65	50	70	60	45
	4	55	45	65	50	45	40
	5	45	60	60	55	75	35
10	6	100	97	105	100	97	65
	7	110	95	80	95	90	65
	8	100	100	85	100	100	70
	9	75	75	80	100	100	40
	10	100	95	110	95	80	55
15	11	205	150	200	170	150	80
	12	120	110	120	125	110	70
	13	120	145	125	130	150	65
	14	125	120	130	135	120	75
	15	130	125	170	115	120	55
18	16	225	200	215	200	220	80
	17	220	235	250	230	240	95
	18	205	215	185	245	220	65
	19	210	180	215	195	200	70
	20	235	220	210	240	210	105
20	21	285	315	275	330	305	125
	22	310	250	330	265	285	115
	23	260	275	235	320	230	95
	24	290	230	240	285	195	110
	25	335	285	310	275	245	130

scheduling problems. It is crucial to assess the trade-off between computation time and the size of the scheduling problem, in order to ensure a practical implementation. Overall, the MILP model provides excellent performance for small to moderate scheduling problems, delivering optimal solutions within a reasonable computational time frame.

## 5. Discussion

The formulated MILP model exhibits limitations regarding the computation time necessary for solving problems with more than 20 jobs. Nonetheless, the model's application can be instrumental in evaluating the performance of metaheuristic solution techniques. This approach allows determining the gap between the optimal solution obtained with the MILP formulation and that achieved by the proposed algorithm. Consequently, this enables the calibration and scaling of the proposed algorithms to accommodate a larger number of jobs, with an initial comparison conducted on a smaller scale.

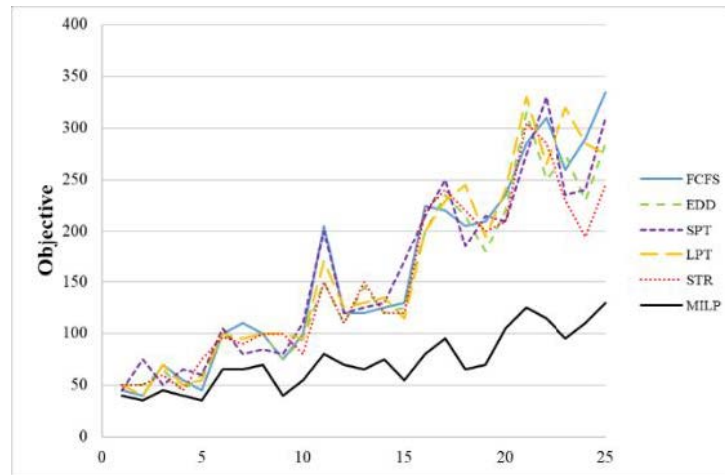


Figure 7. Performance with  $\alpha_j Z_j$  and  $\beta_j Y_j$

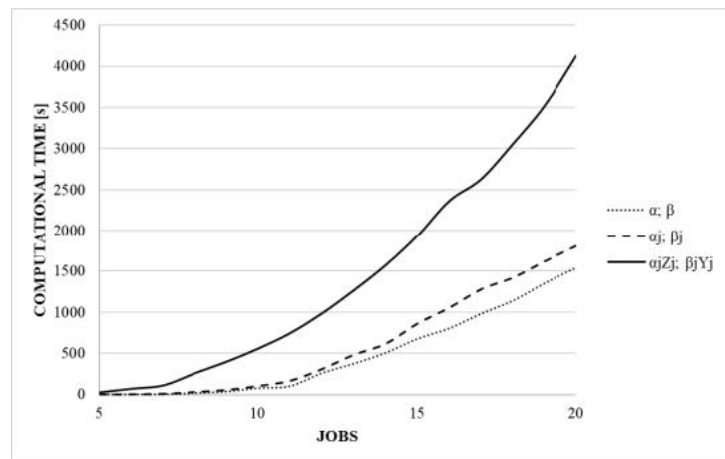


Figure 8. Computation time to solve MILP model vs. the number of jobs

Following the model evaluation, it became apparent that Eq. 12 significantly contributes to the model's complexity, thereby increasing computation times. To address this issue, linearizing the calculation is proposed, introducing the variable  $S_i$ , which determines the start time of the job at position  $i$ . This variable is then incorporated into Eq. 13 to calculate the variable  $C_i$ . This modification results in an approximately 20% reduction in computation times. While this adjustment improves computational efficiency, its impact is not highly significant, and the formulation remains efficient for up to 20 jobs because, when the number of jobs grows, the number of equations also grows.

$$\sum_{j=1}^N (P_j \cdot X_{ji}) + S_{i-1} \leq S_i \quad \text{for all } j \quad (12)$$

$$C_i \geq S_i + B(X_{ji} - 1) \quad \text{for all } j, \text{ for all } i \quad (13)$$

The MILP formulation is very efficient when jobs with variable penalty rates are to be scheduled, which generates a program where the cell incurs a waiting time, so that the jobs do not start before those

required to minimize earliness. Undoubtedly, this characteristic considerably improves performance when evaluating the earliness/tardiness of jobs. However, these waiting times can be considered idle and penalize the cell utilization indicator. This opens the possibility of formulating a bio-objective MILP model, where, in addition to minimizing the earliness/tardiness, cell utilization can be maximized, or the makespan can be minimized.

## 6. Conclusions

The evaluation results emphasize the importance of utilizing advanced mathematical models (e.g., MILP) for optimizing job scheduling in complex systems. While the dispatch rules may perform well for smaller job sets, their performance deteriorates as the complexity and scale of the problem increase.

The model was developed for an assembly cell with a single robot. We recommend adapting the mathematical model to schedule jobs in assembly cells with more than one robot, analyzing the assignment of tasks to robots, transfer times, and flow shop or parallel systems.

The model is efficient when a reduced number of jobs must be scheduled; if there are more jobs, more computational time will be required to solve the model at an exponential rate.

## 7. CRediT author statement

**John Andrés Muñoz Guevara:** conceptualization, investigation, methodology, software, validation, writing (original draft), writing (review & editing).

**Jairo Alberto Villegas Flórez:** methodology, software, validation, writing (review & editing).

**Jhannier Jhoan Jaramillo Tabima:** methodology, validation, writing (review & editing).

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