

Research paper

Calibration of an External Pressure Cell Coupled to a Digital Densimeter for Measuring Liquid Densities

Calibración de una célula de presión externa acoplada a un densímetro digital para medir densidades de líquidos

[José Julio Pinheiro dos Santos Junior](#)¹, [Roberto Guimarães Pereira](#)^{2*}, Dalni Malta do Espírito Santo Filho³, José Mauricio Gomes Gouveia³, [Mila Rosendahl Avelino](#)³

¹[Federal Institute of Rio de Janeiro](#) – IFRJ, Rua Sebastião Lacerda, s/n, CEP: 26600-000, Paracambi - RJ, Brazil,

²[Fluminense Federal University](#) – TEM, PGMEC, PGEB – UFF, Rua Passo da Pátria 156, CEP 24210-240, Niterói - RJ, Brazil, ³[National Institute of Metrology](#), Quality and Technology – Inmetro, Av. Nossa Senhora das Graças 50, Xerém, CEP 25250-000, Duque de Caxias - RJ, Brazil

*Correspondence E-mail: robertogp@id.uff.br

Received: January 10th, 2025

Modified: March 24th, 2026

Accepted: June 17th, 2026

Abstract

Context: The accurate determination of density is crucial because density is an important property in the custody transfer of fluids and in other applications, such as the characterization of fuels and biofuels.

Method: The calibration of the pressure cell, which is connected to the digital densimeter, was performed according to the measurement standards of the Fluids Laboratory of the Instituto Nacional de Metrologia, Qualidade e Tecnologia (Inmetro). This calibration involved determining an internal adjustment factor, which was obtained at each calibration temperature, from 277.15 K to 313.15 K, over a pressure range from 0.1 MPa to 15 MPa. Additionally, reference solutions were used under the same temperature and pressure conditions, as specified by the operational requirements.

Results: The relative expanded uncertainty associated with the calibration curve is 0.00030 g/cm³, for a coverage factor of 2.058 (95% confidence level). It applies to a density range from 0.7033 g/cm³ to 1.0064 g/cm³, a temperature range from 277.15 K to 313.15 K, and a pressure range from 0.1 MPa to 15.0 MPa.

Conclusions: With the pressure cell and all its instruments properly calibrated, this system can be used for a variety of applications within a temperature range from 277.15 K to 313.15 K and a pressure range from 0.1 MPa to 15.0 MPa, including pycnometer calibration, the measurement of liquid densities from 0.7033 g/cm³ to 1.0064 g/cm³, and the calibration of other pressure cells.

Keywords: density, pressure, temperature, uncertainty.

Resumen

Contexto: La determinación exacta de la densidad es crucial, ya que la densidad es una propiedad importante en la transferencia de custodia de fluidos y en otras aplicaciones, como la caracterización de combustibles y biocombustibles.

Método: La calibración de la celda de presión, que está conectada al densímetro digital, se realizó de acuerdo con los patrones de trabajo del Laboratorio de Fluidos del Instituto Nacional de Metrologia, Qualidade e Tecnologia (Inmetro). Esta calibración implicó determinar un factor de ajuste interno, que se obtuvo en cada temperatura de calibración, desde 277,15 K hasta 313,15 K, en un rango de presión de 0,1 MPa a 15 MPa. Además, se utilizaron soluciones de referencia en las mismas condiciones de temperatura y presión, tal como lo especifican los requisitos operacionales.

Resultados: La incertidumbre expandida relativa asociada con la curva de calibración es de 0,00030 g/cm³, para un factor de cobertura de 2,058 (nivel de confianza del 95 %). Se aplica a un rango de densidad de 0,7033 g/cm³ a 1,0064 g/cm³, un rango de temperatura de 277,15 K a 313,15 K y un rango de presión de 0,1 MPa a 15,0 MPa.

Conclusiones: Con la celda de presión y todos sus instrumentos debidamente calibrados, este sistema puede utilizarse para diversas aplicaciones dentro de un rango de temperatura de 277,15 K a 313,15 K y un rango de presión de 0,1 MPa a 15,0 MPa, entre ellas la calibración de picnómetros, la medición de la densidad de líquidos de 0,7033 g/cm³ a 1,0064 g/cm³ y la calibración de otras celdas de presión.

Palabras clave: densidad, incertidumbre, presión, temperatura.

1. Introduction

This research consists of the development of a measurement system for calibrating digital densimeters with pressure cells in order to determine the density of liquids at different temperatures and pressures. The system was developed at the Fluids Laboratory (Laflu) of Inmetro, the Brazilian national metrology institute (NMI), with the aim of calibrating pressure cells used by companies in the petroleum sector. The measurement system uses a steel pycnometer manufactured at Laflu [24], specially designed to contain the liquid at different pressures and temperatures, coupled to the following basic equipment: a digital densimeter with a pressure cell, a pressurization system, and a temperature control system. Together, these form an integrated system that allows density readings to be taken under simultaneous changes in pressure and temperature, at a much lower cost than conventional systems. All equipment used in the measurement system is traceable to national standards, ensuring reliable and metrologically traceable measurement results. This new setup allows the results of the digital densimeter with a pressure cell to be compared with those of steel pycnometers, and the measurement uncertainty of both results to be estimated. With the digital densimeter, curves can be obtained for various fuels relating density,

pressure, and temperature. A digital densimeter was used to measure the density of liquids at various temperatures and pressures. In this process, a steel tube filled with the sample of interest vibrates perpendicular to its axis within an electromagnetic field. The harmonic oscillation frequency of the tube is directly related to the density of the liquid inside it. The density, liquid temperature, liquid pressure, and oscillation period are displayed on the monitor of the host digital densimeter, such as the DMA 4500 M or DMA 5000 M, both manufactured by Anton Paar. The calibration of digital densimeters follows the ISO 15212-1 and ISO 15212-2 standards. However, the external measuring cells used for pressure–volume–temperature (PVT) analysis and liquid density require a calibration methodology different from the one specified in these standards for other digital densimeters. It is important to emphasize that determining the internal factor of the measuring cell with only two fluids, such as water and air, is not sufficient. To calibrate these cells, as well as digital densimeters, traceable reference fluids must be used within the temperature and pressure range over which the measuring cell or digital densimeter is to be calibrated. In this case, the following fluids were used: water, ethanol, n-nonane, and n-dodecane.

Many fluids can be used to determine density; among them, fuels (alcohol, gasoline, diesel, and biodiesel) are of strategic importance, given their role in the Brazilian energy matrix. In particular, biodiesel, which has recently been blended with commercial diesel, is a biofuel of great importance for Brazil. In addition, some alkanes exhibit good stability in density, surface tension, and viscosity, and can therefore be used as certified reference materials (CRMs) for the calibration of important instruments such as tensiometers, digital densimeters (process and bench), and viscometers (capillary and rotational).

The density of the fuels of interest and of some alkane-type reference materials can be obtained with the developed system under the specified temperature and pressure conditions. A specific methodology was developed for measuring density with this system. In addition, the specific objectives are:

- Provision of algorithms to correct for the effects of temperature and pressure on the density of the fluids under study.
- Determination of curves of the thermal expansion coefficient and the compressibility factor of the fluids under study for the pressure and temperature range investigated.
- Calibration of digital densimeters with pressure cells at different temperatures and pressures.
- Support for the custody transfer process in the oil and natural gas industries for the commercialization of their products.
- Support for the Brazilian Association of Technical Standards (ABNT), Brazil's national standardization body, in the preparation of tables of density as a function of temperature and pressure, in line with the International Organization for Standardization (ISO).

The remainder of this paper is organized as follows. Section 2 presents the measurement system developed for density determination under controlled temperature and pressure conditions, including the instrumentation and experimental setup. Section 3 describes the calibration procedure of the external pressure cell, covering the determination of the internal adjustment factor, the verification of the

temperature sensor, and the characterization of the reference liquids. Section 4 presents the mathematical model developed for the calibration curve, while Section 5 describes the uncertainty analysis associated with the proposed methodology. Section 6 presents the validation of the calibration model through experimental measurements and a statistical evaluation of the results obtained. Section 7 discusses the main applications of the calibrated system for density measurements under varying temperature and pressure conditions. Finally, Section 8 summarizes the main conclusions of this work.

2. Measurement System

A system [1] was developed at Inmetro to determine the density of liquids at various temperatures and pressures (Figure 1). The system consists of the following equipment: a comparator pump with a maximum pressure capacity of 100 MPa; a digital densimeter (DMA 5000 M, manufactured by Anton Paar; see Figure 2) with a resolution of 0.000001 g/cm³; an external pressure cell (DMA HP, manufactured by Anton Paar; see Figure 2) capable of operating at pressures up to 70 MPa and temperatures from -10 °C to 300 °C; and a steel pycnometer manufactured at Laflu with a nominal volume of 100 cm³.

3. Calibration Procedure

The calibration of the pressure cell (DMA HP), which is connected to the digital densimeter as shown in Figure 1, was performed according to the working standards of Laflu. This calibration involved determining an internal adjustment factor, which was obtained at each calibration temperature, from 4 °C to 40 °C, over a pressure range from 0.1 MPa to 70 MPa. Additionally, reference solutions were used under the required calibration conditions, within a temperature range from 4 °C to 40 °C and a pressure range from 0.1 MPa to 15 MPa, as specified by the operational requirements.

The pressure cell calibration procedure was carried out as follows:

- Determination of the internal factor of the digital densimeter for a temperature range from 4 °C to 40 °C and a pressure range from 0.1 MPa to 15.0 MPa, and of the constants A and B of the measuring cell;
- Verification of the internal temperature sensor of the pressure cell;
- Measurement of the density of reference solutions over the selected temperature and pressure range;
- Correlation of the density values obtained in the measurement (indicated values) with the density values calculated from the equations derived from the pycnometer measurements.

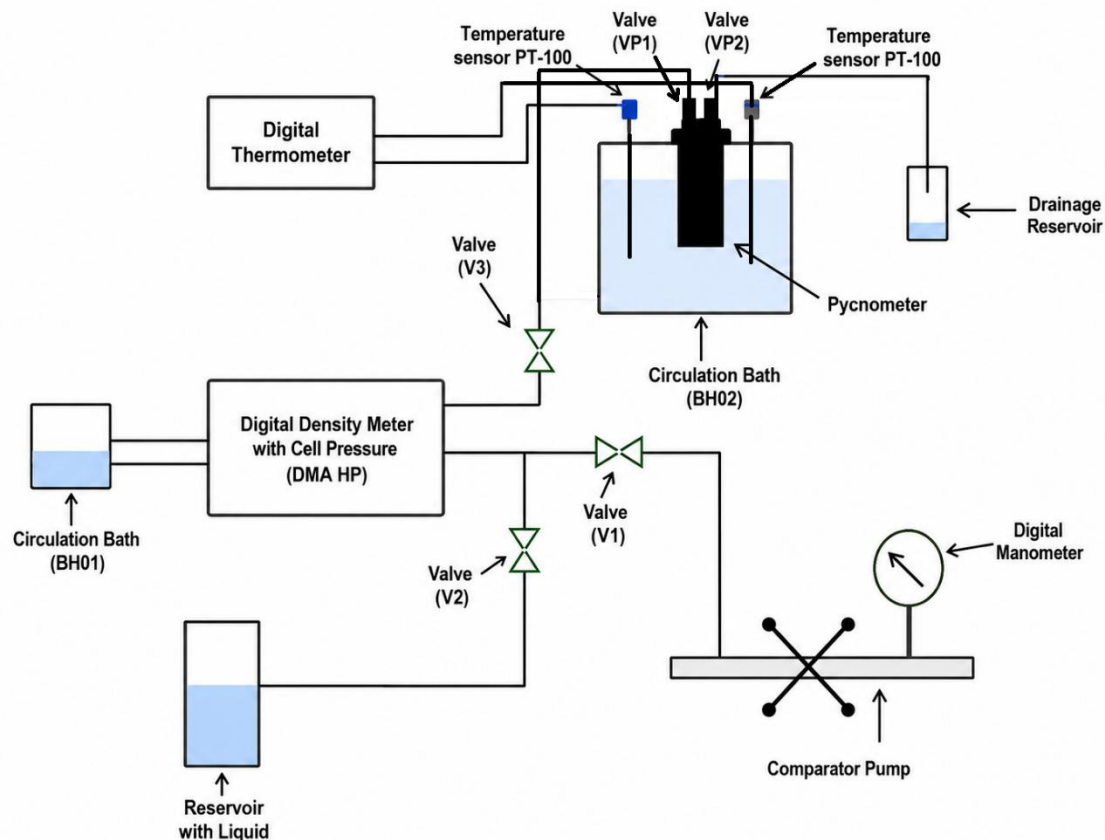


Figure 1. Schematic diagram of the measurement system. Source: Inmetro/Laflu.

3.1. Determination of the Internal Adjustment Factor

To verify the linearity of the measuring cell, an internal adjustment was required. This internal adjustment (internal factor) [2]–[4] was performed using two reference fluids: double-distilled, deaerated water and air. These fluids were chosen because their density values at different temperatures are well known.

To determine the internal adjustment parameters A and B, which characterize the calibration of the pressure cell at a given temperature, the following procedure was used. First, the period and density of air at the specified temperature were measured. Then, approximately 5.0 ml of water was introduced, and the period and density of the water were measured at the same temperature. It is important to note that both fluids were maintained at 0.1 MPa during this process.

The temperatures selected for determining the adjustment parameters A and B were 4 °C, 15 °C, 20 °C, 25 °C, and 40 °C.

The adjustment parameters A and B were calculated using Equations (1) and (2), respectively.

$$A = \frac{\rho_1 - \rho_2}{t_1^2 - t_2^2} \quad (1)$$

$$B = \frac{t_2^2 \cdot \rho_1 - t_1^2 \cdot \rho_2}{t_1^2 - t_2^2} \quad (2)$$

Where:

ρ_1 is the air density (g/cm³);

ρ_2 is the density of double-distilled, deaerated water (g/cm³);

t_1 is the air oscillation period (μs);

t_2 is the double-distilled and deaerated water oscillation period (μs).



Figure 2. Pressure cell (DMA HP) and digital densimeter (DMA 5000M) (Source: Inmetro/Laflu)

3.2 Internal Temperature Sensor Verification

To verify the internal sensor, a PT-100 temperature sensor (standard) with a resolution of 0.001°C (Model: CKT100, Manufacturer: Anton Paar) was placed inside the measuring cell. The temperature reading from the PT-100 sensor was compared with the temperature indicated by the pressure cell. This comparison allowed for the formulation of an equation to correct the temperature reading from the pressure cell. The temperature inside the cell was checked at the following points: 4 °C, 15 °C, 20 °C, 25 °C, and 40 °C.

Equation 3 provides the temperature correction for the liquid during pressure cell measurements.

$$T_{Lc} = b_0 + b_1 \cdot T_{Li} \quad (3)$$

Where:

T_{Lc} is the corrected temperature of the liquid inserted into the pressure cell at °C;

T_{Li} is the temperature of the liquid indicated on the pressure cell in °C;

b_0 and b_1 are constants obtained through the linear regression method.

3.3 Determination of the specific mass of reference liquids at different temperatures and pressures

A steel pycnometer [5], developed at the Inmetro Laboratory, was used to determine the density of reference liquids at various temperatures.

The densities of the reference liquids water; ethanol; n-nonane, and n-dodecane [6] were defined for temperature and pressure ranges from 4°C to 40°C and 0.1 MPa to 15 MPa. The equations used to determine the densities of these liquids [1],[7],[8] are provided below.

Table I
Equations to determine the densities of reference liquids

Reference Liquids	Mathematical Model
Water	$\frac{\rho_{LR}(T_L, P_L)}{\rho_0(T_r, P_r)} = n_0 + n_1 \cdot \left(\frac{T_L}{T_0} - 1\right) + n_2 \cdot \left(\frac{P_L}{P_0} + 1\right) + n_3 \cdot \left(\frac{T_L}{T_0} - 1\right)^2 \quad (4)$
Ethanol, N-Dodecane/ N-Nonane	$\frac{\rho_{LR}(T_L, P_L)}{\rho_0(T_r, P_r)} = n_0 + n_1 \cdot \left(\frac{T_L}{T_0} - 1\right) + n_2 \cdot \left(\frac{P_L}{P_0} + 1\right) + n_3 \cdot \left(\frac{T_L}{T_0} - 1\right)^2 + n_4 \cdot \left(\frac{P_L}{P_0} + 1\right)^2 \quad (5)$

where

$\rho_{LR}(T_L, P_L)$ is the density of the reference liquids at temperature T_L and pressure P_L within the working range in g/cm³;

$\rho_0(T_r, P_r)$ is the density of the reference liquids obtained under reference condition ($T_r = 20$ °C and $P_r = 0,1$ MPa) through measurements performed by the hydrostatic weighing system developed at the Fluids Laboratory of Inmetro [1] in g/cm³;

T_0 is the initial temperature = 4 °C

P_0 is the initial pressure = 0.1 MPa

T_L is the temperature of the liquid in °C;

P_L is the pressure of the liquid in MPa;

n_0, n_1, n_2, n_3 e n_4 are constants obtained through the multiple linear regression method (Table II);

Table II
Constants of equations 4 and 5

Constants of Equations (4) and (5)	Reference Solutions			
	Water	Ethanol	N-Dodecane	N-Nonane
n_0	1.001895	1.016023	1.015689	1.016774
n_1	-1.7197×10^{-4}	-3.9755×10^{-3}	-3.7363×10^{-3}	-4.1966×10^{-3}
n_2	4.5784×10^{-5}	1.2685×10^{-4}	9.0507×10^{-5}	1.1454×10^{-4}
n_3	-8.1428×10^{-5}	-2.0696×10^{-5}	-4.5816×10^{-6}	-4.0948×10^{-6}
n_4	-	-1.2905×10^{-7}	-4.7947×10^{-8}	-4.8400×10^{-8}

Tables III, IV, V and VI represent the uncertainty tables used in determining the expanded uncertainty of the specific mass of the reference liquids through equations (4) and (5) and the contributions of each input quantity in determining these uncertainties, respectively.

Table III
Uncertainty budget for the measurement of water density obtained with Equation (4)

Input quantity (X_i)	Standard uncertainty $u(X_i)$	Distribution type	Sensitivity coefficient (C_j)	Uncertainty contribution $u_i(y)$ (g/cm^3)
$\rho_{L\text{med.}}$	$9.500 \times 10^{-5} \text{ g}/\text{cm}^3$	Student's t	1	9.500×10^{-5}
ρ_0	$2.50 \times 10^{-6} \text{ g}/\text{cm}^3$	Student's t	0.9918	2.479×10^{-6}
T_L	$5.00 \times 10^{-4} \text{ }^\circ\text{C}$	normal	$-8.128 \times 10^{-4} \text{ g}/(\text{cm}^3 \cdot ^\circ\text{C})$	-4.064×10^{-7}
P_L	$4.50 \times 10^{-3} \text{ MPa}$	normal	$1.627 \times 10^{-4} \text{ g}/(\text{cm}^3 \cdot \text{MPa})$	7.323×10^{-7}
$\delta(\rho_L)$	$6.880 \times 10^{-5} \text{ g}/\text{cm}^3$	rectangular	1	6.879×10^{-5}

Note: $\rho_{L\text{med}}$ is the density of the liquid measured with the pycnometer [1].

Table IV
Uncertainty budget for the measurement of ethanol density obtained with Equation (5)

Input quantity (X_i)	Standard uncertainty $u(X_i)$	Distribution type	Sensitivity coefficient (C_j)	Uncertainty contribution $u_i(y)$ (g/cm^3)
$\rho_{L\text{med.}}$	$1.250 \times 10^{-4} \text{ g}/\text{cm}^3$	Student's t	1	1.250×10^{-4}
ρ_0	$1.00 \times 10^{-5} \text{ g}/\text{cm}^3$	normal	0.9948	9.948×10^{-6}
T_L	$5.00 \times 10^{-4} \text{ }^\circ\text{C}$	normal	$-8.598 \times 10^{-4} \text{ g}/(\text{cm}^3 \cdot ^\circ\text{C})$	-4.299×10^{-7}
P_L	$4.50 \times 10^{-3} \text{ MPa}$	normal	$6.951 \times 10^{-4} \text{ g}/(\text{cm}^3 \cdot \text{MPa})$	3.128×10^{-6}
$\delta(\rho_L)$	$6.154 \times 10^{-5} \text{ g}/\text{cm}^3$	rectangular	1	6.154×10^{-5}

Table V

Uncertainty budget for the measurement of n-dodecane density obtained with Equation (5)

Input quantity (X_i)	Standard uncertainty $u(X_i)$	Distribution type	Sensitivity coefficient (C_i)	Uncertainty contribution $u_i(y)$ (g/cm^3)
$\rho_{\text{Lmed.}}$	$9.500 \times 10^{-5} \text{ g}/\text{cm}^3$	Student's t	1	9.500×10^{-5}
ρ_0	$3.00 \times 10^{-6} \text{ g}/\text{cm}^3$	Student's t	1.025	3.074×10^{-6}
T_L	$5.00 \times 10^{-4} \text{ }^\circ\text{C}$	normal	$-7.149 \times 10^{-4} \text{ g}/(\text{cm}^3 \cdot ^\circ\text{C})$	-3.574×10^{-7}
P_L	$4.50 \times 10^{-3} \text{ MPa}$	normal	$5.693 \times 10^{-4} \text{ g}/(\text{cm}^3 \cdot \text{MPa})$	2.562×10^{-6}
$\delta(\rho_L)$	$7.661 \times 10^{-5} \text{ g}/\text{cm}^3$	rectangular	1	7.661×10^{-5}

Table VI

Uncertainty budget for the measurement of n-nonane density obtained with Equation (5)

Input quantity (X_i)	Standard uncertainty $u(X_i)$	Distribution type	Sensitivity coefficient (C_i)	Uncertainty contribution $u_i(y)$ (g/cm^3)
$\rho_{\text{Lmed.}}$	$9.500 \times 10^{-5} \text{ g}/\text{cm}^3$	Student's t	1	9.500×10^{-5}
ρ_0	$3.50 \times 10^{-6} \text{ g}/\text{cm}^3$	Student's t	1.029	3.601×10^{-6}
T_L	$5.00 \times 10^{-4} \text{ }^\circ\text{C}$	normal	$-7.664 \times 10^{-4} \text{ g}/(\text{cm}^3 \cdot ^\circ\text{C})$	-3.832×10^{-7}
P_L	$4.50 \times 10^{-3} \text{ MPa}$	normal	$7.173 \times 10^{-4} \text{ g}/(\text{cm}^3 \cdot \text{MPa})$	3.228×10^{-6}
$\delta(\rho_L)$	$8.571 \times 10^{-5} \text{ g}/\text{cm}^3$	rectangular	1	8.571×10^{-5}

Table VII presents the relative expanded uncertainties associated with these measurements, obtained with Equations (4) and (5) for the different liquids over a temperature range from 4 °C to 40 °C and a pressure range from 0.1 MPa to 15.0 MPa. These results were obtained for a confidence level of approximately 95%.

Table VII

Relative expanded uncertainties of the densities of the reference liquids obtained with Equations (4) and (5)

Reference liquid	Relative expanded uncertainty U_{rel} (%)	Coverage factor (k)	Effective degrees of freedom (ν_{eff})
Water	0.024	2.043	60
Ethanol	0.036	2.064	41
n-Dodecane	0.033	2.036	71
n-Nonane	0.036	2.030	86

4. Mathematical Model for the Pressure Cell Calibration Curve

To develop this mathematical model, it was necessary to record the density values of the reference liquids indicated by the pressure cell at temperatures of 4 °C, 15 °C, 20 °C, 25 °C, and 40 °C, and at pressures of 0.1 MPa, 2.5 MPa, 5.0 MPa, 7.5 MPa, 10.0 MPa, and 15.0 MPa. These experimental values were then compared with the density values calculated from the equations shown in Table I under the same temperature and pressure conditions. Using the experimental data obtained from the measurements, Equation (6) was determined by multiple linear regression:

$$\rho_{Lc} = d_0 + d_1 \cdot \rho_{Li} + d_2 \cdot \rho_{Li}^2 + d_3 \cdot T_{Li} + d_4 \cdot T_{Li}^2 + d_5 \cdot P_L^2 + d_6 \cdot P_L^4 + (d_7 \cdot T_{Li} + d_8 \cdot \rho_{Li}) \cdot P_L^2 \quad (6)$$

where

ρ_{Lc} is the corrected density of the liquids, in g/cm³;

ρ_{Li} is the density of the reference liquids indicated on the digital densimeter, in g/cm³;

T_{Li} is the temperature of the liquid indicated on the digital densimeter, in °C;

P_L is the pressure exerted on the liquid inside the pressure cell and in the system, in MPa;

$d_0, d_1, d_2, d_3, d_4, d_5, d_6, d_7, d_8$ are constants obtained by multiple linear regression.

5. Uncertainty Analysis of the Density Obtained with the Pressure Cell Calibration Curve

The input variables considered for the density equation of the liquid measured with the pressure cell are as follows:

$$\rho_{Lc} = \Phi(T_{Li}, P_L, \rho_{LR}, R, \delta(\rho_{Lc})) \quad (7)$$

where

T_{Li} is the temperature of the liquid indicated on the digital densimeter;

P_L is the pressure of the liquid in the system;

ρ_{LR} is the density of the reference liquids determined from Equations (4) and (5), in g/cm³;

R is the system resolution, in g/cm³;

$\delta(\rho_{Lc})$ is the maximum deviation between the density of the liquid determined with Equation (6) and the density of the liquid obtained with Equations (4) and (5).

For the uncertainty analysis of this measurand, the following steps are necessary [9], [10]:

- Analysis of the standard uncertainty of the liquid temperature (T_L):

$$u(T_L) = \sqrt{\left(\frac{U_{cert.}(T_L)}{2}\right)^2 + \left(\frac{d_T}{2\sqrt{3}}\right)^2} \quad (8)$$

where

$U_{cert.}(T_L)$ is the expanded uncertainty of the temperature sensor obtained from the calibration certificate;

d_T is the resolution of the temperature sensor.

- Analysis of the standard uncertainty of the pressure exerted on the liquid (P_L):

$$u(P_L) = \sqrt{\left(\frac{U_{cert.}(P_L)}{2}\right)^2 + \left(\frac{d_P}{2\sqrt{3}}\right)^2} \quad (9)$$

where

$U_{cert.}(P_L)$ is the expanded uncertainty of the pressure sensor obtained from the calibration certificate;

d_P is the resolution of the pressure sensor.

- Analysis of the standard uncertainty of the liquid density obtained from Equations (4) and (5):

$$u(\rho_L(T_L, P_L)) = \frac{U(\rho_L(T_L, P_L))_{max.}}{2} \quad (10)$$

where

$U(\rho_L(T_L, P_L))_{max.}$ is the maximum expanded uncertainty obtained for the density of the liquid determined with Equations (4) and (5).

- Analysis of the standard uncertainty of the pressure cell resolution (R):

$$u(R) = \frac{r}{2\sqrt{3}} \quad (11)$$

where

r is the resolution of the pressure cell, in this case 0.00001 g/cm³.

- Analysis of the standard uncertainty of the maximum deviation of Equation (6), $\delta(\rho_{Lc})$:

$$u(\delta(\rho_{Lc})) = \frac{\delta_{max}(\rho_{Lc})}{2\sqrt{3}} \quad (12)$$

$$\delta_{max}(\rho_{Lc}) = \rho_{Lc} - \rho_L(T_L, P_L) \quad (13)$$

where

ρ_{Lc} is the density of the reference liquid obtained with Equation (6);

$\rho_L(T_L, P_L)$ is the density of the liquid obtained with Equation (4) or (5);

$\delta_{max}(\rho_{LC})$ is the maximum deviation between the results of Equation (6) and those of Equations (4) and (5).

Determination of the sensitivity coefficients for these input variables:

$$C(T_{Li}) = d_3 + 2 \cdot d_4 \cdot T_{Li} + d_7 \cdot P_L^2 \quad (14)$$

$$C(P_L) = 2 \cdot d_5 \cdot P_L + 4 \cdot d_6 \cdot P_L^3 + 2 \cdot (d_7 \cdot T_{Li} + d_8 \cdot \rho_{Li}) \cdot P_L \quad (15)$$

The sensitivity coefficients of the liquid density obtained from Equations (4) and (5), the pressure cell resolution (R), and the maximum deviation from Equation (4), $\delta(\rho_{LC})$, are equal to 1.

The variance of the density can then be written as follows:

$$u_{\rho_{LC}}^2 = C(T_{Li})^2 \cdot u(T_{Li})^2 + C(P_L)^2 \cdot u(P_L)^2 + C(\rho_{LR})^2 \cdot u(\rho_{LR})^2 + C(R)^2 \cdot u(R)^2 + C(\delta(\rho_{LC}))^2 \cdot u(\delta(\rho_{LC}))^2 \quad (16)$$

The expanded uncertainty [10] of the reference liquid density measured with the pressure cell is defined by the following equation:

$$U(\rho_{LC}) = k \cdot u_{\rho_{LC}} \quad (17)$$

where

$u_c(\rho_{LC})$ is the combined standard uncertainty of the density of the reference liquids obtained with the pressure cell;

k is the coverage factor for a confidence level of approximately 95%.

6. Validation

Table VIII presents the parameters required to determine the internal adjustment factors of the DMA HP pressure cell at the following temperatures: 4 °C, 15 °C, 20 °C, 25 °C, and 40 °C. These parameters were established at a pressure of 0.1 MPa. Parameters A and B are defined by Equations (1) and (2), respectively.

Table VIII
Internal adjustment parameters of the pressure cell

Reference temperature T_r (°C)	t_1 (μs)	ρ_1 (g/cm ³)	t_2 (μs)	ρ_2 (g/cm ³)	A (g/(cm ³ ·μs ²))	B (g/cm ³)
4.000	2415.84	0.001276	2627.81	0.999972	9.340037×10^{-7}	5.449865
10.000	2417.69	0.001246	2629.84	0.999699	9.323671×10^{-7}	5.448699
15.000	2419.25	0.001222	2631.45	0.999099	9.310185×10^{-7}	5.447832
18.000	2420.27	0.001208	2632.38	0.998595	9.303251×10^{-7}	5.448002
20.000	2420.80	0.001199	2632.99	0.998203	9.297482×10^{-7}	5.447409
22.000	2421.42	0.001190	2633.61	0.997769	9.291240×10^{-7}	5.446610
25.000	2422.36	0.001177	2634.47	0.997044	9.285175×10^{-7}	5.447289
30.000	2423.94	0.001156	2635.96	0.995646	9.269993×10^{-7}	5.445470
40.000	2427.10	0.001115	2638.79	0.992212	9.241109×10^{-7}	5.442613

Note: ρ_1 is the density of air; ρ_2 is the density of double-distilled, deaerated water; t_1 is the oscillation period of air; t_2 is the oscillation period of double-distilled, deaerated water.

Table IX presents the values required to determine the constants defined in Equation (3), which are used to correct the temperature of the liquid measured in the pressure cell.

Table IX
Temperatures indicated by the pressure cell and by the standard temperature sensor

T_{io} (°C)	T_{ip} (°C)	Correction = $T_{ip} - T_{io}$ (°C)
4.006	4.798	0.792
4.006	4.797	0.791
15.003	15.537	0.534
15.004	15.464	0.460
18.003	18.467	0.464
18.014	18.218	0.204
20.002	20.317	0.315
20.001	20.151	0.150
22.005	22.183	0.178
22.005	22.183	0.178
25.002	25.076	0.074
25.002	25.076	0.074
30.008	29.869	-0.139
30.008	29.870	-0.138
39.999	39.510	-0.489
39.999	39.510	-0.489

Note: T_{io} is the temperature indicated by the pressure cell, and T_{ip} is the temperature indicated by the standard temperature sensor inserted into the pressure cell.

The constants b_0 and b_1 of the adjustment curve (Figure 3), obtained by linear regression, are presented in Table X.

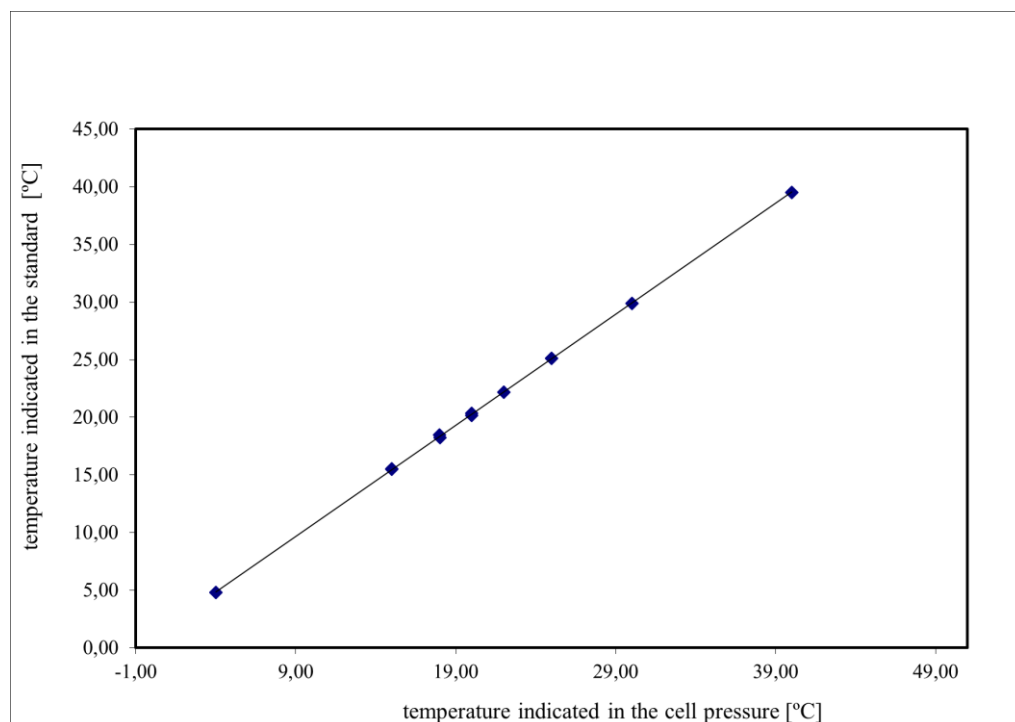


Figure 3. Adjustment curve of the temperature indicated by the standard sensor as a function of the temperature indicated by the pressure cell.

Table X

Constants of Equation (3)

Constant	Value
b_0	0.9799
b_1	0.9635

Using the experimental data presented in Tables XI, XII, XIII, and XIV for each reference liquid, Equation (6) was determined by multiple linear regression to establish the pressure calibration curve of the cell at temperatures of 4 °C, 15 °C, 20 °C, 25 °C, and 40 °C (277.15 K, 288.15 K, 293.15 K, 298.15 K, and 313.15 K, respectively) and at pressures of 0.1 MPa, 2.5 MPa, 5.0 MPa, 7.5 MPa, 10.0 MPa, and 15.0 MPa. The density values of the reference liquids were obtained using a pycnometer during the density measurements.

Table XI

**Density values of n-nonane indicated by the pressure cell (ρ_{Li}) and reference density values ($\rho_{Lref.}$) [11]
under the same temperature and pressure conditions**

T_{Li} (°C)	P_{Li} (MPa)	ρ_{Li} (g/cm ³)	$\rho_{Lref.}$ (g/cm ³)
3.995	0.10	0.73015	0.73010
15.000	0.10	0.72169	0.72179
20.001	0.10	0.71816	0.71800
24.999	0.10	0.71407	0.71420

40.005	0.10	0.70274	0.70274
3.994	2.50	0.73231	0.73205
15.000	2.50	0.72402	0.72374
20.002	2.50	0.72037	0.71995
24.998	2.50	0.71646	0.71615
40.003	2.50	0.70476	0.70469
3.998	5.00	0.73440	0.73404
15.000	5.00	0.72619	0.72573
20.002	5.00	0.72197	0.72194
24.998	5.00	0.71881	0.71814
40.002	5.00	0.70726	0.70668
3.998	7.50	0.73651	0.73598
15.000	7.50	0.72837	0.72768
20.002	7.50	0.72391	0.72388
24.997	7.50	0.72014	0.72009
40.003	7.50	0.70885	0.71780
3.999	10.00	0.73771	0.73789
15.000	10.00	0.72952	0.72958
20.003	10.00	0.7258	0.72579
24.997	10.00	0.72208	0.72199
40.003	10.00	0.71096	0.71053
3.999	15.00	0.74106	0.74156
15.000	15.00	0.73379	0.73325
20.002	15.00	0.72944	0.72946
24.999	15.00	0.72593	0.72566
40.002	15.00	0.71420	0.71420

Table XII

**Density values of n-dodecane indicated by the pressure cell (ρ_{Li}) and reference density values ($\rho_{Lref.}$) [12]
under the same temperature and pressure conditions**

T_{Li} (°C)	P_{Li} (MPa)	ρ_{Li} (g/cm ³)	$\rho_{Lref.}$ (g/cm ³)
3.992	0.10	0.76028	0.76068
14.999	0.10	0.75296	0.75296
20.002	0.10	0.74943	0.74943
25.001	0.10	0.74589	0.74590
40.009	0.10	0.73521	0.73521
3.991	2.50	0.76219	0.76229
14.999	2.50	0.75432	0.75456
20.002	2.50	0.75081	0.75103
25.002	2.50	0.74736	0.74750
40.004	2.50	0.73649	0.73682
3.992	5.00	0.76413	0.76391
15.001	5.00	0.75633	0.75619
20.002	5.00	0.75288	0.75266
25.002	5.00	0.74949	0.74912
40.004	5.00	0.73869	0.73845
3.999	7.50	0.76607	0.76549
15.001	7.50	0.75850	0.75777
20.003	7.50	0.75491	0.75424
25.005	7.50	0.75070	0.75070
40.004	7.50	0.74003	0.74003
3.999	10.00	0.76760	0.76702
15.001	10.00	0.75978	0.75930
20.003	10.00	0.75590	0.75577
25.004	10.00	0.75273	0.75224

40.001	10.00	0.74224	0.74156
4.001	15.00	0.76996	0.76996
15.001	15.00	0.76224	0.76224
20.003	15.00	0.75871	0.75871
25.003	15.00	0.75517	0.75517
40.002	15.00	0.74450	0.74450

Table XIII

**Density values of ethanol indicated by the pressure cell (ρ_{Li}) and reference density values ($\rho_{Lref.}$) [13], [14]
under the same temperature and pressure conditions**

T_{Li} (°C)	P_{Li} (MPa)	ρ_{Li} (g/cm ³)	$\rho_{Lref.}$ (g/cm ³)
4.010	0.10	0.80450	0.80382
15.003	0.10	0.79523	0.79506
20.002	0.10	0.79089	0.79099
24.999	0.10	0.78667	0.78687
39.999	0.10	0.77425	0.77420
4.010	2.50	0.80636	0.80568
15.003	2.50	0.79719	0.79692
20.002	2.50	0.79289	0.79285
24.998	2.50	0.78873	0.78873
39.997	2.50	0.77579	0.77607
4.000	5.00	0.80909	0.80848
15.003	5.00	0.80005	0.79971
20.002	5.00	0.79582	0.79564
24.997	5.00	0.79173	0.79152
39.997	5.00	0.77900	0.77886
4.000	7.50	0.81087	0.81067
15.003	7.50	0.80192	0.80189
20.002	7.50	0.79773	0.79782
24.997	7.50	0.79369	0.79371
39.997	7.50	0.78110	0.78104
4.000	10.00	0.81348	0.81272
15.003	10.00	0.80467	0.80395
20.002	10.00	0.80054	0.79988
24.999	10.00	0.79655	0.79576
39.996	10.00	0.78329	0.78310
4.000	15.00	0.81717	0.81645
15.000	15.00	0.80836	0.80768
20.000	15.00	0.80423	0.80361
25.000	15.00	0.80024	0.79949
40.000	15.00	0.78786	0.78683

Table XIV

**Density values of water indicated by the pressure cell (ρ_{Li}) and reference density values ($\rho_{Lref.}$) [15]
under the same temperature and pressure conditions**

T_{Li} (°C)	P_{Li} (MPa)	ρ_{Li} (g/cm ³)	$\rho_{Lref.}$ (g/cm ³)
3.996	0.10	0.99987	1.00019
14.993	0.10	0.99926	0.99910
20.005	0.10	0.99821	0.99820
25.004	0.10	0.99714	0.99705

40.004	0.10	0.99215	0.99206
3.996	2.50	1.00145	1.00129
14.993	2.50	1.00073	1.00020
20.004	2.50	0.99976	0.99930
25.006	2.50	0.99857	0.99814
40.002	2.50	0.99325	0.99316
3.997	5.00	1.00243	1.00243
15.000	5.00	1.00140	1.00134
20.005	5.00	1.00040	1.00044
25.005	5.00	0.99925	0.99928
40.002	5.00	0.99436	0.99430
3.998	7.50	1.00360	1.00357
15.003	7.50	1.00250	1.00248
20.005	7.50	1.00160	1.00158
24.998	7.50	1.00040	1.00043
40.001	7.50	0.99544	0.99544
3.997	10.00	1.00480	1.00471
14.996	10.00	1.00370	1.00363
20.004	10.00	1.00270	1.00272
24.996	10.00	1.00150	1.00157
40.002	10.00	0.99652	0.99658
3.996	15.00	1.00720	1.00700
14.998	15.00	1.00600	1.00591
20.005	15.00	1.00490	1.00501
24.998	15.00	1.00370	1.00386
40.004	15.00	0.99865	0.99887

The values of the constants $d_0, d_1, d_2, d_3, d_4, d_5, d_6, d_7, d_8$ are shown in Table XV.

Table XV
Values of the constants of Equation (6)

Constant	Value
d_0	0.009163 g/cm ³
d_1	0.977794
d_2	0.013037 (g/cm ³) ⁻¹
d_3	2.09237×10^{-6} g/(cm ³ ·°C)
d_4	-4.91365×10^{-9} g/(cm ³ ·°C ²)
d_5	-3.68076×10^{-6} g/(cm ³ ·MPa ²)
d_6	1.35105×10^{-8} g/(cm ³ ·MPa ⁴)
d_7	-6.54144×10^{-8} g/(cm ³ ·MPa ² ·°C)
d_8	1.87645×10^{-6} g/(cm ³ ·MPa ²)

Table XVI presents the uncertainty budget used to determine the expanded uncertainty associated with the pressure cell calibration curve obtained with Equation (6), along with the contribution of each input quantity to this uncertainty.

Table XVI
Uncertainty budget for the pressure cell calibration curve

Input quantity (X_i)	Standard uncertainty $u(X_i)$	Distribution type	Sensitivity coefficient (C_i)	Uncertainty contribution $u_i(y)$ (g/cm ³)
T_{Li}	4.101×10^{-2} °C	normal	2.678×10^{-5} g/(cm ³ ·°C)	1.10×10^{-6}
P_L	1.500×10^{-2} MPa	normal	-4.128×10^{-7} g/(cm ³ ·MPa)	-6.19×10^{-9}

$\rho_{Lref.}$	$1.397 \times 10^{-4} \text{ g/cm}^3$	Student's t	1	1.40×10^{-4}
R	$2.887 \times 10^{-6} \text{ g/cm}^3$	rectangular	1	2.89×10^{-6}
$\delta(\rho_{LC})$	$-2.871 \times 10^{-5} \text{ g/cm}^3$	rectangular	1	-2.87×10^{-5}

The relative expanded uncertainty associated with the calibration curve of Equation (6) is 0.00030 g/cm^3 , for a coverage factor of 2.058 (95% confidence level). It applies to a density range from 0.7033 g/cm^3 to 1.0064 g/cm^3 , a temperature range from 277.15 K to 313.15 K , and a pressure range from 0.1 MPa to 15.0 MPa .

To evaluate the quality of the adopted calibration curve (Equation (6)), the reduced chi-square (χ_v^2) [5] was used.

The results shown in Table XVII indicate that χ_v^2 is less than 1, confirming that Equation (6), adopted as the calibration curve of the DMA HP pressure cell, is appropriately defined under the calibration conditions. Therefore, Equation (6) can be used to determine the density of various liquids from 0.7033 g/cm^3 to 1.0064 g/cm^3 , at temperatures from $4 \text{ }^\circ\text{C}$ to $40 \text{ }^\circ\text{C}$ (277.15 K to 313.15 K) and at pressures up to 15 MPa , with an expanded uncertainty of 0.00030 g/cm^3 .

Table XVII
Parameters for calculating the reduced chi-square (χ_v^2) of the calibration curve adopted for the pressure cell

Parameter	Value
χ^2	533.3948
Number of measurements (N)	600
Degrees of freedom (v)	599
χ_v^2	0.9

To verify the reliability of the density results obtained with Equation (6) for the liquids measured in the pressure cell, the normalized error (E_n) was calculated [23]. The E_n values were evaluated according to the following criteria: if $|E_n| \leq 1$, the values are considered compatible; if $|E_n| > 1$, the values are not compatible, and corrective action is required for the measurement. In this analysis, density results for the liquids reported in the reference articles [19]–[22] were used as reference values. The normalized error was calculated with the following equation:

$$E_n = \frac{\rho_{LC} - \rho_{ref}}{\sqrt{U(\rho_{LC})^2 + U(\rho_{ref})^2}} \quad (18)$$

where ρ_{LC} is the density of the liquid obtained with Equation (6), ρ_{ref} is the density of the liquid reported in the reference articles, $U(\rho_{LC})$ is the expanded uncertainty associated with this measurement, and $U(\rho_{ref})$ is the expanded uncertainty associated with the reference density. The normalized errors for the analyzed temperature and pressure conditions are presented in Tables XVIII, XIX, XX, and XXI.

Table XVIII

Evaluation of the liquid density results obtained with Equation (6) against the reference values at 20 °C and 0.1 MPa

Reference	Liquid	ρ_{LC} (g/cm ³)	$U(\rho_{LC})$ (g/cm ³)	ρ_{ref} (g/cm ³)	$U(\rho_{ref})$ (g/cm ³)	E_n
[19]	n-Nonane	0.71854	0.00030	0.71815	0.00040	0.78
[20]	n-Dodecane	0.74931	0.00030	0.7496	0.0015	-0.15
[21]	Ethanol	0.79069	0.00030	0.7896	0.0024	0.48
[22]	Water	0.99824	0.00030	0.99821	0.00001	0.09

Table XIX

Evaluation of the liquid density results obtained with Equation (6) against the reference values at 20 °C and 5.0 MPa

Reference	Liquid	ρ_{LC} (g/cm ³)	$U(\rho_{LC})$ (g/cm ³)	ρ_{ref} (g/cm ³)	$U(\rho_{ref})$ (g/cm ³)	E_n
[19]	n-Nonane	0.72185	0.00030	0.72209	0.00040	-0.47
[20]	n-Dodecane	0.75265	0.00030	0.7530	0.0015	-0.22
[21]	Ethanol	0.79553	0.00030	0.7938	0.0024	0.73
[22]	Water	1.00037	0.00030	1.00040	0.00001	-0.11

Table XX

Evaluation of the liquid density results obtained with Equation (6) against the reference values at 40 °C and 0.1 MPa

Reference	Liquid	ρ_{LC} (g/cm ³)	$U(\rho_{LC})$ (g/cm ³)	ρ_{ref} (g/cm ³)	$U(\rho_{ref})$ (g/cm ³)	E_n
[19]	n-Nonane	0.70281	0.00030	0.70235	0.00040	0.92
[20]	n-Dodecane	0.73521	0.00030	0.7348	0.0015	0.31
[21]	Ethanol	0.77411	0.00030	0.7722	0.0023	0.81
[22]	Water	0.99218	0.00030	0.99222	0.00001	-0.13

Table XXI

Evaluation of the liquid density results obtained with Equation (6) against the reference values at 40 °C and 5.0 MPa

Reference	Liquid	ρ_{LC} (g/cm ³)	$U(\rho_{LC})$ (g/cm ³)	ρ_{ref} (g/cm ³)	$U(\rho_{ref})$ (g/cm ³)	E_n
[19]	n-Nonane	0.70722	0.00030	0.70679	0.00040	0.86
[20]	n-Dodecane	0.73851	0.00030	0.7365	0.0015	-0.07
[21]	Ethanol	0.77874	0.00030	0.77695	0.0023	0.76
[22]	Water	0.99431	0.00030	0.99436	0.00001	-0.18

The normalized errors range from -0.47 to 0.92, as presented in Tables XVIII, XIX, XX, and XXI, indicating compatibility between the density values obtained with Equation (6) and the reference values.

7. Applications

The calibration methodology developed in this work can be applied to several activities involving density measurements under controlled temperature and pressure conditions. One important application is the calibration of steel pycnometers used to determine the density of liquids under pressure. Since the proposed system provides traceable density measurements with known uncertainty, it can be used as a reference system for pycnometer calibration.

Another important application is the determination of liquid density over a wide range of temperatures and pressures. This capability is particularly useful for fuels, biofuels, petroleum products, and other industrial fluids whose density varies significantly with thermodynamic conditions.

The calibrated pressure cell may also be used for the calibration and verification of other pressure cells and density measurement systems operating under similar conditions. In this context, the methodology contributes to establishing the metrological traceability and comparability of measurement results.

In addition, the proposed system can support activities in the petroleum and energy sectors, including custody transfer operations, process control, and the characterization of fuels and biofuels. Reliable density measurements under pressure are essential for volume correction procedures, inventory control, and commercial transactions involving liquid products.

Therefore, the methodology presented in this work extends the applicability of digital densimeters equipped with external pressure cells and provides a practical tool for metrology laboratories and industrial users who require accurate density measurements under varying pressure and temperature conditions.

8. Conclusions

With the pressure cell and all its instruments properly calibrated, this system can be used for a variety of applications within a temperature range from 4 °C to 40 °C (277.15 K to 313.15 K) and a pressure range from 0.1 MPa to 15.0 MPa, including pycnometer calibration, the measurement of liquid densities from 0.7033 g/cm³ to 1.0064 g/cm³, and the calibration of other pressure cells.

The normalized errors (En) of the liquid density measurements obtained with Equation (6) lie between -1 and 1, which indicates that the results are compatible with the reference values.

It is worth emphasizing that the accurate determination of density is crucial because density is an important property in the custody transfer of fluids and in other applications, such as the characterization of fuels and biofuels [8], [16]–[18].

Acknowledgements

R. G. Pereira thanks the National Council for Scientific and Technological Development (CNPq), and J. J. P. Santos Junior thanks PRONAMETRO for financial support. The authors also thank the National Institute of Metrology, Quality and Technology (Inmetro) for providing the laboratory infrastructure used to conduct this research.

Author Contributions

José Julio Pinheiro dos Santos Junior: Conceptualization, Methodology, Investigation, Formal analysis, Data curation, Writing – original draft.

Roberto Guimarães Pereira: Conceptualization, Supervision, Writing – Review & Editing.

Mila Rosendahl Avelino: Validation, Formal analysis, Writing – Review & Editing.

Dalni Espirito Santo Filho: Validation, Writing – Review & Editing.

José Mauricio Gomes Gouveia: Resources, Writing – Review & Editing.

Access to Research Data

The data supporting the findings of this study are available from the corresponding author upon reasonable request.

Conflict of Interest

The authors declare that they have no competing interests.

Statement on Artificial Intelligence

During the preparation of this manuscript, the authors used ChatGPT (OpenAI) to assist with language revision, text editing, and improvement of the manuscript's organization and readability. After using this tool, the authors carefully reviewed and edited the content as necessary and take full responsibility for the accuracy and integrity of the published work.

References

- [1] J. J. P. Santos Junior, Development of a System for Determining the Density of Liquids as a Function of Simultaneous Pressure and Temperature Variation, Ph.D. dissertation, Universidade Federal Fluminense, Niterói, Brazil, 2020 (in Portuguese).
- [2] Oscillation-Type Density Meters—Part 1: Laboratory Instruments, ISO 15212-1:1998, International Organization for Standardization, Geneva, Switzerland, 1998.
- [3] Oscillation-Type Density Meters—Part 2: Process Instruments for Homogeneous Liquids, ISO 15212-2:2002, International Organization for Standardization, Geneva, Switzerland, 2002.
- [4] Guidelines on the Calibration of Oscillation-Type Density Meters, SIM MWG7/cg-02/v.00, Sistema Interamericano de Metrología (SIM), 2016.
- [5] S. Lorefice, R. Romeo, M. Santiano, and A. Capelli, “Original Pycnometers for Volatile Liquid Density over Wide Ranges of Temperature and Pressure: Practical Example,” *Metrologia*, vol. 51, no. 3, pp. 154–250, 2014, <https://doi.org/10.1088/0026-1394/51/3/154>
- [6] J. Díaz, L. Becerra, L. Centeno, F. Castillo, and U. Bravo, “Desarrollo de materiales de referencia certificados en densidad,” in Proc. Simposio de Metrología, Mexico, 2010.
- [7] J. J. P. Santos Junior et al., “Measurements and Correlations of Density, Isothermal Compressibility Factor, and Thermal Expansion Coefficient of Anhydrous Ethanol Fuel as a Function of Temperature and Pressure,” *Int. J. Thermophys.*, vol. 42, Art. no. 78, 2021, <https://doi.org/10.1007/s10765-021-02825-w>
- [8] J. J. P. Santos Junior, R. G. Pereira, M. Rosendahl, A. J. S. M. de Mendonça, D. M. do Espirito Santo Filho, and J. M. Gouveia, “Determination of Density, Isobaric Thermal Expansivity Coefficient and Isothermal Compressibility Coefficient Correlations for n-Dodecane and n-Nonane as a Function of Temperature and Pressure,” *Int. J. Thermophys.*, vol. 43, Art. no. 107, 2022, <https://doi.org/10.1007/s10765-022-03024-x>.
- [9] Evaluation of Measurement Data—Guide to the Expression of Uncertainty in Measurement, JCGM 100:2008, Joint Committee for Guides in Metrology, 2008.
- [10] Expression of the Uncertainty of Measurement in Calibration, EA-4/02, 1st Brazilian ed. Rio de Janeiro, Brazil: INMETRO, ABNT, and SBM, 1999 (in Portuguese).

- [11] G. Schilling, R. Kleinrahm, and W. Wagner, “Measurement and Correlation of the (p, ρ , T) Relation of Liquid n-Heptane, n-Nonane, 2,4-Dichlorotoluene, and Bromobenzene in the Temperature Range from 233.15 K to 473.15 K at Pressures up to 30 MPa for Use as Density Reference Liquids,” *J. Chem. Thermodyn.*, vol. 40, no. 7, pp. 1095–1105, 2008, <https://doi.org/10.1016/j.jct.2008.02.020>
- [12] M. L. Singh, S. C. Tripathi, P. P. K. Venkata, and V. G. Gaikar, “Correlations Among Composition, Temperature, and Density, Viscosity, or Derived Thermodynamic Properties of Binary Mixtures of Tri-n-butyl Phosphate with n-Hexane or n-Dodecane,” *Ind. Eng. Chem. Res.*, vol. 53, pp. 3795–3804, 2014, <https://doi.org/10.1021/ie4036912>
- [13] Y. Takiguchi and M. Uematsu, “Densities for Liquid Ethanol in the Temperature Range from 310 K to 480 K at Pressures up to 200 MPa,” *J. Chem. Thermodyn.*, vol. 28, no. 1, pp. 7–16, 1996, <https://doi.org/10.1006/jcht.1996.0003>
- [14] H. Bettin and F. Spieweck, “A Revised Formula for the Calculation of Alcoholometric Tables,” *PTB-Mitteilungen*, vol. 100, no. 6, pp. 457–460, 1990.
- [15] Alcoholometric Tables, ABNT NBR 5992:2008, 2nd ed. Rio de Janeiro, Brazil: Associação Brasileira de Normas Técnicas, 2008 (in Portuguese).
- [16] D. Maestre, S. Orjuela, and J. Duarte, “Experimental Assessment of the Emissions Characteristics of Low Displacement Diesel Engines Operating with Biodiesel Blends from Algae Oil,” *Ingeniería*, vol. 27, no. 2, Art. no. e17539, 2022, <https://doi.org/10.14483/23448393.17539>
- [17] D. Herrera, J. Bermúdez, and C. Castilla, “Análisis del desempeño de la potencia y el torque de un motor diésel operando con mezclas de biodiésel de palma,” *Ingeniería*, vol. 25, no. 3, pp. 250–263, 2020, <https://doi.org/10.14483/23448393.15676>.
- [18] L. Mónico, J. Sandoval, and A. Rodríguez, “Estudio teórico sobre la influencia del uso de mezclas de biodiésel de aceite de palma con Jet A-1 en motores a reacción,” *Ingeniería*, vol. 22, no. 1, pp. 140–151, 2017, <https://doi.org/10.14483/udistrital.jour.reving.2017.1.a06>
- [19] E. W. Lemmon and R. Span, “Short Fundamental Equations of State for 20 Industrial Fluids,” *J. Chem. Eng. Data*, vol. 51, no. 3, pp. 785–850, 2006, <https://doi.org/10.1021/je050186n>.
- [20] E. W. Lemmon and M. L. Huber, “Thermodynamic Properties of n-Dodecane,” *Energy Fuels*, vol. 18, no. 4, pp. 960–967, 2004, <https://doi.org/10.1021/ef0341062>.
- [21] J. A. Schroeder, S. G. Penoncello, and J. S. Schroeder, “A Fundamental Equation of State for Ethanol,” *J. Phys. Chem. Ref. Data*, vol. 43, Art. no. 043102, 2014, <https://doi.org/10.1063/1.4895394>
- [22] W. Wagner and A. Pruss, “The IAPWS Formulation 1995 for the Thermodynamic Properties of Ordinary Water Substance for General and Scientific Use,” *J. Phys. Chem. Ref. Data*, vol. 31, no. 2, pp. 387–535, 2002, <https://doi.org/10.1063/1.1461829>

- [23] M. G. Cox, “The Evaluation of Key Comparison Data: Determining the Largest Consistent Subset,” *Metrologia*, vol. 44, pp. 187–200, 2007, <https://doi.org/10.1088/0026-1394/44/3/005>.
- [24] J. J. P. Santos Junior et al., “The Use of Ethanol to Determine the Volume of a Pycnometer Used to Measure the Density of Liquids at Different Temperatures and Pressures,” *J. Braz. Soc. Mech. Sci. Eng.*, vol. 42, Art. no. 50, 2020, <https://doi.org/10.1007/s40430-019-2107-y>

Authors and biographies

José Julio Pinheiro dos Santos Junior

He received the B.Sc. and M.Sc. degrees in Mechanical Engineering from the Pontifical Catholic University of Rio de Janeiro (PUC-Rio), Rio de Janeiro, Brazil, in 1994 and 1998, respectively, and the Ph.D. degree in Mechanical Engineering from the Fluminense Federal University (UFF), Niterói, Brazil, in 2020. He is a Professor and Researcher at the Federal Institute of Rio de Janeiro (IFRJ), Rio de Janeiro, Brazil. His research interests include thermodynamics, volume and density measurements, calibration procedures, digital densimeters, and numerical methodologies applied to metrology. Email: jose.pinheiro@ifrj.edu.br

Roberto Guimarães Pereira

He received the B.Sc. degree in Mechanical Engineering from Gama Filho University, Rio de Janeiro, Brazil, in 1982, and the M.Sc. and Ph.D. degrees in Mechanical Engineering from the Pontifical Catholic University of Rio de Janeiro (PUC-Rio), Rio de Janeiro, Brazil, in 1985 and 1992, respectively. He is a retired Full Professor in the Department of Mechanical Engineering at the Fluminense Federal University (UFF), Niterói, Brazil, and is currently a Professor in the Graduate Programs in Mechanical Engineering and Biosystems Engineering at UFF. His research interests include rheology, energy and the environment, rheological and physicochemical characterization of materials, biofuels, biomass and waste utilization, solar energy, and energy generation and emissions evaluation using biofuels. Email: robertogp@id.uff.br

Dalni Malta do Espírito Santo Filho

He received the B.Sc. degree in Mechanical Engineering from the State University of Rio de Janeiro (UERJ), Rio de Janeiro, Brazil, in 1994 the M.Sc. degree in Mechanical Engineering from the Federal University of Rio de Janeiro (COPPE/UFRJ), Rio de Janeiro, Brazil, in 1998, and the Ph.D. degree in Mechanical Engineering from the Fluminense Federal University (UFF), Niterói, Brazil, in 2010. He is a Researcher-Technologist at the National Institute of Metrology, Quality and Technology (Inmetro), Rio de Janeiro, Brazil. His research interests include density, viscosity, surface tension, volume, measurement uncertainty, calibration, and fluid metrology. He served as Inmetro’s representative on the

Bureau International des Poids et Mesures (BIPM) Working Groups on Density and Viscosity. Email: dsfilho@inmetro.gov.br

José Mauricio Gomes Gouveia

He received the B.Sc. degree in Mechanical Engineering and the M.Sc. degree in Nuclear Engineering from the Federal University of Rio de Janeiro (UFRJ), Rio de Janeiro, Brazil, in 1987 and 2000, respectively. He is a Researcher at the National Institute of Metrology, Quality and Technology (Inmetro), Rio de Janeiro, Brazil, where he currently serves as Head of the Fluids Laboratory (Laflu). His work focuses on fluid metrology, calibration, and the development of measurement systems and metrological procedures for fluid-related quantities. Email: jmgouveia@inmetro.gov.br

Mila Rosendahl Avelino

She received the B.Sc., M.Sc., and Ph.D. degrees in Mechanical Engineering from the Federal University of Rio de Janeiro (PEM/COPPE/UFRJ), Rio de Janeiro, Brazil, in 1994, 1996, and 2000, respectively. She also completed a specialization in Experimental Fluid Mechanics at the University of Surrey, Guildford, United Kingdom. She has been a Professor at the State University of Rio de Janeiro (UERJ), Rio de Janeiro, Brazil, since 1994, and a Researcher at the National Institute of Metrology, Quality and Technology (Inmetro) since 2011. From 2000 to 2004, she served as a NATO expert and as a Researcher at the University of Miami. In 2010, she received the International Centre for Heat and Mass Transfer (ICHMT) Hartnett–Irvine Medal for her contributions to microchannel research. Email: mrosendahl@inmetro.gov.br