Voltage Stability and Jacobian Determinant in Power Systems: Analysis, Indicators, and Practical Monitoring Tools

Voltage stability is a fundamental aspect of power system reliability, which refers to an electrical network's ability to maintain acceptable voltage levels under normal operating conditions and various types of disturbances [1]. This involves complex, nonlinear power flow equations that capture the intricate relationships between bus voltages, phase angles, and power injections across the system [2]. As demand and system conditions change, maintaining voltage stability becomes increasingly challenging, and failure to do so can lead to voltage collapse and widespread blackouts [3]. Therefore, understanding and monitoring the the factors that influence voltage stability are crucial for ensuring the secure, reliable, and efficient operation of modern power grids.

1 Power flow equations

Consider a power system with n buses. The complex power injected at bus i is $S_i = P_i + jQ_i$, which is related to the bus voltage $V_i\angle\delta_i$ and the admittance matrix Y_{ij}. The active and reactive power flow equations are presented below [4].

$$P_i = V_i \sum_{j=1}^{n} V_j \left(G_{ij}\cos(\delta_i - \delta_j) + B_{ij}\sin(\delta_i - \delta_j)\right), \ \forall i \in \mathcal{N} \qquad (1)$$

$$Q_i = V_i \sum_{j=1}^{n} V_j \left(G_{ij}\sin(\delta_i - \delta_j) - B_{ij}\cos(\delta_i - \delta_j)\right), \ \forall i \in \mathcal{N} \qquad (2)$$

where:

- V_i and δ_i are the magnitude and angle of the voltage at bus i,
- G_{ij} and B_{ij} represent the conductance and susceptance elements of the admittance matrix Y_{ij}.
- $\mathcal{N}$ denotes the set containing all the nodes in the transmission network.

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1.1 Jacobian matrix and voltage stability

The power flow equations can be vectorially expressed as follows [5]:

$$\mathbf{F}(\mathbf{x}) = 0,$$

where $\mathbf{x} = [\delta_2, \ldots, \delta_n, V_2, \ldots, V_n]^T$.

Linearizing around an operating point yields

$$\Delta\mathbf{F} = \mathbf{J}\Delta\mathbf{x},$$

where $\mathbf{J}$ is the Jacobian matrix, which comprises the partial derivatives

$$\mathbf{J} = \begin{bmatrix} \frac{\partial P}{\partial \delta} & \frac{\partial P}{\partial V} \\ \frac{\partial Q}{\partial \delta} & \frac{\partial Q}{\partial V} \end{bmatrix}.$$

2 Voltage stability and Newton-Raphson power flow divergence

The stability of a power system as it approaches its operating limits is closely associated with the behavior of the Newton-Raphson power flow method. Under normal conditions, this iterative technique converges rapidly to a solution, indicating a stable operating point [5]. However, as the system load increases toward its stability boundary, the power flow solutions tend to become more sensitive, and the Newton-Raphson method may fail to converge [6].

This divergence in the power flow solution is a practical indicator that the system is nearing a voltage collapse or severe voltage instability. This occurs when the Jacobian matrix $\mathbf{J}$, derived from the linearization of the nonlinear power flow equations, approaches singularity [3]. As the load increases, $|\mathbf{J}|$, *i.e.*, the determinant of the Jacobian, tends to decrease, reflecting the system's diminishing ability to maintain voltage stability.

When $|\mathbf{J}|$ approaches zero, the power flow equations become unsolvable with the iterative method, signifying that the system is at or beyond its stability margin [7]. This non-convergence signals that the system cannot support further load increases without risking voltage collapse.

Monitoring the convergence behavior of the power flow alongside the evolution of the Jacobian determinant provides valuable early warning signals. With this information, operators can implement preventive control actions—such as reactive power compensation or system reconfiguration—in order to avoid instability and ensure a secure operation.

Remark 1 *A divergence of the Newton-Raphson power flow method indicates that* $|\mathbf{J}| \to 0$*, and that a voltage collapse may occur, highlighting the critical link between solution convergence, the Jacobian determinant, and voltage stability. Recognizing these signs early allows for timely interventions aimed at preventing voltage collapse and maintaining system reliability [1].*

The next section presents a simple yet effective MATLAB/Matpower tool to evaluate this behavior, facilitating a practical assessment of system stability limits.

3 Matpower tool for monitoring power flow divergence under load increases

To practically demonstrate how increasing system load impacts the solvability of power flow equations and, consequently, a system's voltage stability, a MATLAB/Matpower code is presented below [8]. This tool incrementally raises the load in a predefined case (*e.g.*, *case9*) and monitors the success or failure of the power flow solution at each step. Failure to converge indicates a potential approach to the system's stability limit, often associated with voltage collapse.

```
clc;
step = 0.001;
max_load_multiplier = 5;
start_multiplier = 1;  % Assuming original load is at 1
% Loop over load multipliers
load_multiplier = start_multiplier;
for k = start_multiplier:step:max_load_multiplier
    % Create a copy to avoid modifying original case
    % Scale load (columns 3 and 4 of bus matrix: Pd and Qd)
    mpc = loadcase('case9');  % Load the case
    mpc.bus(:, 3:4) = k * mpc.bus(:, 3:4);
    % Run power flow once per load level
    results = runpf(mpc);
    if results.success
        disp(['Success at load multiplier: ', num2str(k)]);
        % Optional: Store results if needed
    else
        disp(['Failure at load multiplier: ', num2str(k-step)]);
        break;
    end
end
```

This analysis revolves around the use of a straightforward yet powerful method to determine the load levels at which power flow equations lose solvability. These points are critical since they indicate an approach to voltage instability, which is a major concern in power system operation. Operators can utilize these insights for preventive measures, thereby ensuring grid reliability.

4 Numerical assessment

Table II summarizes the maximum loadability limits for several IEEE test systems:

Table II: Maximum loadability for IEEE feeders

IEEE system	Maximum load multiplier	Approx. loadability (% of original value)
IEEE 9-Bus	2.373	237.3
IEEE 14-Bus	4.004	400.4
IEEE 30-Bus	3.657	365.7
IEEE 57-Bus	1.785	178.5
IEEE 118-Bus	1.816	181.6

4.1 Results analysis

- **Maximum load multiplier.** This value indicates how many times the original load can be handled by the system before becoming unstable.
  - For example, the IEEE 9-bus system can sustain approximately 2.373 times its initial load, while the IEEE 14-bus system can handle about 4.004 times.
  - Larger systems like the IEEE 118-bus feeder support up to approximately 1.816 times the original load, reflecting different stability margins across system sizes.
- **Approximate loadability (% of the original load).** This value represents the maximum load that the system can support relative to its initial load.
  - The IEEE 14-bus system exhibits the highest loadability (400.4%), indicating a significant load margin.
  - Conversely, the IEEE 57-bus feeder's maximum loadability is about 178.5%, suggesting a more constrained margin.

4.2 Main findings

- **Loadability variations across systems.** Smaller systems like the IEEE 9-bus and 14-bus feeders have a higher relative load capacity compared to larger systems such as the IEEE 57-bus grid, highlighting differences regarding their inherent system stability.
- **Practical implications.** Load multipliers provide crucial thresholds for operators to prevent voltage collapse. When the load approaches these limits, system reinforcement or control actions become necessary.
- **System size and stability margin.** Generally, larger systems tend to exhibit a lower loadability, which underscores the importance of robust stability monitoring in large interconnected grids.

5 Conclusion

Analyzing a power system's maximum loadability and the behavior of the Jacobian determinant provides crucial insights with regard to voltage stability margins. As the system load increases, the Jacobian matrix approaches singularity, which is signified by the determinant $|\mathbf{J}| \to 0$. This condition indicates that the power flow equations are nearing their solvability limit, and that the system is close to voltage collapse. Monitoring these indicators in real time can serve as an early-warning mechanism, allowing operators to implement preventive control actions such as reactive power support, reconfiguration, or load shedding to maintain stability and avoid catastrophic failures.

Understanding the relationship between the Jacobian determinant and system stability not only aids in operational decision-making; it can also inform system planning processes. By quantifying load margins, utilities can identify vulnerabilities and prioritize infrastructure upgrades or control strategies that effectively enlarge the stability region. Furthermore, incorporating these assessments into operation routines enhances resilience by providing a clear measure of how close the system is to its operating limits, thereby supporting the implementation of proactive measures that help to ensure a reliable and secure power delivery under increasing demand or unforeseen disturbances.

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