Artículo de Investigación

Modeling and parameter calculation of photovoltaic fields in irregular weather conditions

Modelado y cálculo de parámetros de campos fotovoltaicos en condiciones climáticas irregulares

Juan David Bastidas Rodríguez
Universidad del Valle
Escuela de Ingeniería
Eléctrica y Electrónica
juan.d.bastidas@correo.univalle.edu.co

Carlos Andrés Ramos-Paja
Universidad Nacional de Colombia
Sede Medellín
Facultad de Minas
caramosp@unal.edu.co

Edinson Franco Mejía
Universidad del Valle
Escuela de Ingeniería
Eléctrica y Electrónica
edinson.franco@correo.univalle.edu.co

Abstract

In this paper, a model for photovoltaic (PV) fields and a procedure for calculating the parameters of the modules of a string (a group of PV modules connected in series) are presented. The proposed model considers the series and parallel resistors of the photovoltaic modules, the bypass diodes as ideal switches and represents the blocking diodes with the Shockley equation, which allow to express the string voltage as an explicit function of the load current by using the Lambert-W function; in this way it is possible to construct one equation per string for calculating its current for a given voltage. That procedure is repeated for each string and the current of the field is evaluated by adding the currents of all strings. The parameter calculation procedure evaluates the voltage and current of each active module from the current and voltage of the string and fits the current equation of a module to the experimental data. The accuracy of the proposed model was tested experimentally with a string of two modules obtaining an accuracy improvement near to 50%.

Key words: modeling, photovoltaic fields, mismatching.

Resumen

Este artículo se presenta un modelo de campos fotovoltaicos (PV) y un procedimiento para calcular los parámetros de los módulos de una cadena fotovoltaica (grupo de módulos PV conectados en serie). El modelo propuesto considera las resistencias en serie y paralelo de los módulos, los diodos de bypass como interruptores ideales y aproxima los diodos de bloqueo con la ecuación de Shockley, permitiendo expresar el voltaje en función de la corriente (explícitamente) usando la función de Lambert-W; esto hace posible construir una ecuación por cada cadena para calcular su corriente en un voltaje dado. El procedimiento se repite para cada cadena y la corriente del campo se calcula sumando las corrientes de todas las cadenas. Para encontrar los parámetros se calcula el voltaje y la corriente de cada módulo activo a partir de la corriente y el voltaje de la cadena y luego se ajusta la ecuación de corriente de un módulo a los datos experimentales. La precisión del modelo fue probada experimentalmente con una cadena de dos módulos obteniendo una mejora cercana al 50 %.

Palabras clave: modelado, campos fotovoltaicos, condiciones irregulares.

1. Introduction

Nowadays there is a generalized interest in reducing the CO_2 emissions due to the climate change and the fact that a large part of the energy consumed in the world is produced from fossil fuels [1]. That is why the development of renewable energy sources , like the PhotoVoltaic(PV), grows year by year[2]. Countries like Sweden, Japan, Spain, Germany, among others, have implemented programs to incentive large (MW) and small (kW) PV installations [3]. Those programs have increased the number of PV system installed in the rooftops of the buildings in the last few years and the International Energy Agency (IEA) trends show that such a tendency will continue in the next years [2].

A PV array installed in urban environments may operate in irregular weather conditions (mismatching), which produces large drops in the maximum electrical power available. The two principal causes of such conditions are the shadows produced by surrounding objects (trees, buildings, etc.) and the differences between parameters of the PV panels due to manufacture tolerances. Moreover a PV array (various strings connected in parallel) in mismatching conditions exhibits multiple maximum power points, therefore is not simple to calculate the optimal operating conditions for the PV system.

Traditional Maximum Power Point Tracking (MPPT) strategies (P&O, Incremental Conductance, Extremum Seeking Control, etc.) normally consider uniform irradiance conditions, so the mismatching phenomenon makes possible that those strategies force the PV panel operation around a local maximum power instead the global maximum power, producing waste of energy. Moreover, to analyze the economic viability of PV systems, it is necessary to predict the amount of potential energy that the installation can produce in large time periods (months, years, etc.). To afford both cases it is required a realistic PV model that uses a reasonable computational cost, providing both accuracy and calculation speed.

In the literature there are different models for simulating PV systems [4]–[9], but some of them are very complex [4], [9] and the other ones do not consider the mismatching phenomenon[7], [8].The model proposed in [5] considers the standard single-diode model for each PV module and approximates the bypass diode to an ideal switch. The authors proposed to pose one system of n non-linear equation per string for calculating the voltage of the n active modules in the string and then the model calculates the current from the voltage of any PV module. Such a solution provides a high calculation speed but since the adopted single-diode model disregards the series and parallel resistors for each module, it is difficult to ensure the model accuracy for experimental applications with multiple irradiance conditions.

In [4] the authors consider the series and parallel resistors of the single-diode model as well as the Shockley approximation of the bypass and blocking diodes. Given the implicit relation of the voltage and current it is necessary to use the Lambert-W function [10] for writing the current as an explicit function of the voltage. This model has high accuracy for variable irradiance conditions, but given the implicit relations and the existence of $n+1$ non-linear equations based on Lambert-W function, the simulation time is larger in comparison with the model in [5].

Other approach is presented in[11], where the PV field is divided to calculate the strings voltage for a given current. Then, string currents with the same voltage are added

to calculate the array voltage vs. current characteristic; nevertheless this model have some disadvantages with respect to [5] and [4]: it is necessary to pre-calculate the complete characteristics curves of all strings for calculating the PV field behavior for a given voltage, not all the strings voltages coincide hence some errors will occur, and the model parameters of all modules are considered equal.

In this paper the authors propose a PV model considering the series and parallel resistors of the single-diode model, the bypass diode as an ideal switch, and the blocking diode of each string represented by the Shockley model. By using the Lambert-W function[10], the PV module voltage is expressed in function of the PV current. The proposed modeling approach divides the PV field into strings, for each string, one equation is constructed by equating the string voltage (known value) with the sum of *n* modules and the blocking diode voltages. Then, the overall array current is calculated by adding the currents for each string. Such a procedure requires solving a single non-linear equation to calculate each string current instead of solving an *n+1* non-linear equations system with *n+1* unknown variables as in classical PV modeling approaches. Therefore, the proposed model provides a tradeoff between highly accurate (but slow) and fast (but with low accuracy for different irradiance conditions) models. A shorter version of this paper entitled "Modeling of photovoltaic fields in mismatching conditions by means of inflection voltages" [12], appeared in the 2012 IEEE Workshop on Engineering Applications (WEA-2012). In this extended version the authors additionally present a procedure for calculating the parameters of the PV modules from the measurements of the current and voltage of each string by calculating the experimental I vs. V curve of each PV module, and then fitting the explicit expression of the current with the experimental data.

The paper is organized as follows. Section 2 describes the modeling of the PV module, then Section 3 introduces the calculation of the bypass diodes activation voltages. Sections 4 and 5 address the strings and array current calculation, respectively. Section 6 presents the parameters calculation, Section 7 presents the experimental validation of the model and finally, the conclusions close the work.

2. Model of a PV module using Lambert-W function

The single-diode model, shown in Figure 1, is widely adopted in literature for modeling PV cells[1] [8] [11]. The current source represents the electron flow produced by photon collisions in the semiconductor material, and the non-linear behavior of the P-N junction is modeled through a diode in parallel. The losses caused by a leakage current are represented by the parallel resistor and the resistance of the associated to the electric contacts is modeled with the series resistor. By using the Kirchhoff law, Equation (1) is obtained, where I_{cell} and V_{cell} are current and voltage of the cell, respectively, Iph_{cell} is the current produced by the photovoltaic effect (PV current), A_{cell} is the inverse saturation current of the diode, n_{cell} is the ideal factor of the P-N junction, Rs_{cell} and Rh_{cell} are series and parallel resistors, respectively. k is the Boltzmann constant, q is electron charge, and T is the cell temperature in Kelvin degrees. For the sake of simplicity, $B_{cell}=n_{cell}.k.T/q$ is adopted.

$$I_{cell} = Iph_{cell} - A_{cell} \cdot \left(\exp\left(\frac{V_{cell} + I_{cell} \cdot Rs_{cell}}{B_{cell}} \right) - 1 \right) - \frac{V_{cell} + I_{cell} \cdot Rs_{cell}}{Rh_{cell}} \tag{1}$$
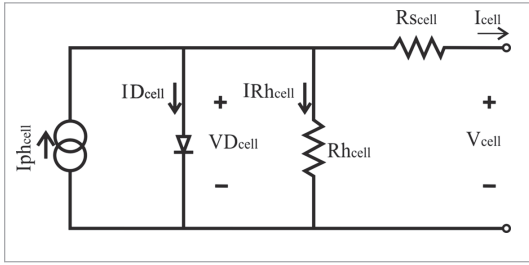
Figure 1. Single-diode model of a PV cell.

A PV module is composed by several parallel cell-strings, each one with a given number of cells connected in series. For a module composed by Np parallel connected strings, where each string has Ns cells in series, the model parameters (A, B, Rs and Rh) and variables (Ipv, Vpv and Iph) are given by Equations (2).

$$Vpv = Ns \cdot V_{cell}$$
$$Ipv = Np \cdot I_{cell}$$
$$Iph = Np \cdot Iph_{cell}$$
$$A = Np \cdot A_{cell}$$
$$B = Ns \cdot B_{cell} \tag{2}$$
$$Rs = \frac{Ns}{Np} \cdot Rs_{cell}$$
$$Rh = \frac{Ns}{Np} \cdot Rh_{cell}$$

The values of A, B, Rs and Rh can be calculated from the datasheet information of a PV panel as shown in [8] [12] and the value of Iph is calculated from irradiance and temperature measurements.

A typical PV module is composed by one string of Ns series-connected cells. Therefore, replacing (2) in (1) with $Np=1$, the current expression becomes Equation (3).

$$Ipv = Iph - A \cdot \left(\exp\left(\frac{Vpv + Ipv \cdot Rs}{B} \right) - 1 \right) - \frac{Vpv + Ipv \cdot Rs}{Rh} \tag{3}$$

There are two parameters that depend on the atmospheric conditions: the inverse saturation current (A), which strongly changes with temperature as defined by Equation (4) [8]; and the photovoltaic current (Iph), which is directly proportional to the irradiance level but also increases, in a small proportion, when the cell temperature rises. Iph behavior is expressed in Equation (5).

$$A = \frac{Isc_{STC} + KIsc \cdot (T - T_{STC})}{\exp\left(\frac{Voc_{STC} + KVoc \cdot (T - T_{STC})}{B} \right) - 1} \tag{4}$$

$$Iph = \frac{S}{S_{STC}} \left(Iph_{STC} + KIsc \cdot (T - T_{STC}) \right) \tag{5}$$

In both equations, Isc_{STC} and Iph_{STC} are the short-circuit current and PV current in Standard Test Conditions (STC), respectively, Voc_{STC} is the open-circuit voltage in STC, $KIsc$ and $KVoc$ are the temperature coefficients of the short-circuit current and open-circuit voltage, respectively, T is the cell temperature, T_{STC} is the cell temperature under STC (both in Kelvin degrees), S is the irradiance and S_{STC} is the STC irradiance.

Multiplying both sides of Equation (3) by Rh/B, defining $\hat{a}$ as in Equation (6), and multiplying the whole expression by $exp(\hat{a})$, Equation (7) is obtained. Note that the right side of the latter does not depend on Vpv.

$$\beta = -Ipv \cdot \frac{Rh}{B} \cdot \left(1 + \frac{Rs}{Rh}\right) + Iph \cdot \frac{Rh}{B} + A \cdot \frac{Rh}{B} - \frac{Vpv}{B} \tag{6}$$

$$\beta \exp(\beta) = \frac{Rh \cdot A}{B} \cdot \exp\left(\frac{Rh}{B} \cdot \left(Iph + A - Ipv\right)\right) \tag{7}$$

Applying the Lambert-W function (denoted by W_0) on both sides of Equation (7), it obtains $\hat{a}$. From the resulting equation it is simple to solve Vpv (PV module voltage (Vpv)) as a function of PV current (Ipv) for a given radiation level S (included in Iph), and temperature T (included in A, B and Iph). The resulting expression is shown in Equation (8).

$$Vpv = -B \cdot W_0\left(\frac{A \cdot Rh}{B} \cdot \exp\left(\frac{Rh}{B} \cdot \left(Iph + A - Ipv\right)\right)\right) - Ipv \cdot \left(Rs + Rh\right) + Rh \cdot \left(Iph + A\right) \tag{8}$$

Commercial PV panels are composed by cells collected in modules, which are groups of cells connected in series with a bypass diode to protect them from damage. Let Ns be the number of cells connected in series to form a module, I_j and $Iph_{i,j}$ represent the output and photovoltaic currents of the i-th module from the j-th string, respectively, so the voltage of a particular module is calculated with Equation (9), that is, assuming that all of its cells are under the same radiation S and temperature T. With a similar procedure it is possible to obtain the module current in function of the voltage given by Equation (10), where $c_1 = 1 + Rs/Rh$.

$$V_{i,j} = -B \cdot W_0\left(\frac{A \cdot Rh}{B} \cdot \exp\left(\frac{Rh}{B} \cdot \left(Iph_{i,j} + A - I_j\right)\right)\right) - I_j \cdot \left(Rs + Rh\right) + Rh \cdot \left(Iph_{i,j} + A\right) \tag{9}$$

$$I_j = -\frac{B}{Rs} \cdot W_0\left(\frac{A \cdot Rs}{c_1 \cdot B} \cdot \exp\left(\frac{V_{i,j} + Rs \cdot \left(A + Iph_{i,j}\right)}{c_1 \cdot B}\right)\right) + \frac{Iph_{i,j} + A - V_{i,j}/Rh}{c_1} \tag{10}$$

3. Calculation of the Inflection voltages

The model is formulated to represent a PV field composed by n modules connected in series and m strings in parallel (Figure 2), where each string has a blocking diode to avoid back flow currents, and each module has a bypass diode to protecting it from hot spot phenomenon. The bypass diodes are modeled as ideal switches for simplifying the calculations; it turns on when the string current is higher than the short circuit current of a particular module ($Isc_{i,j}$), thus the voltage at which this occurs is called *inflection voltage*[5]. Moreover, the proposed modeling approach calculates the PV strings voltages and currents individually, so the analysis of a single string (j-th string) is performed in this section.
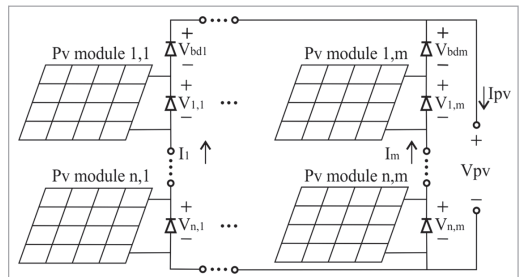

Figure 2. PV Field with m strings of n modules parallel connected.

The first step of the modeling approach is to calculate the inflection voltages, in a similar way as explained in [5], but in this case considering the series and parallel resistors. For a PV string with n modules, and organizing the PV currents in decreasing order, two consecutive modules fulfill $Iph_k > Iph_{k+1}$; the *inflection voltage* ($Vo_{k,k+1}$) corresponds to the minimum string voltage in which the bypass diodes of both consecutive modules are not active (off), therefore both PV modules voltages (Vpv_k and Vpv_{k+1}) are greater than zero, providing energy to the load. Otherwise, when the string voltage is lower than $Vo_{k,k+1}$, Vpv_{k+1} is zero ($k+1$ bypass diode is turned on), then only the k PV modules delivers energy to the load.

In general, the inflection voltages are calculated in the following conditions: $Vpv_{k+1}=0$ and $Ipv_k=Isc_{k+1}$, hence $Vpv_k=Vo_{k,k+1}$. Therefore, $Vo_{k,k+1}$ is the contribution of module k to the inflection voltage of $k+1$ module.

Equation (11) is obtained by replacing $Ipv_k=Isc_{k+1}$ in Equation (9), for calculating $Vo_{k,k+1}$, where $c_2=Rh_k/B_k$ and $I_{Tk}=Iph_k+A_k$. Such a voltage only depends on model parameters of k-th module (Rs_k and Rh_k), cell temperature (A_k and B_k are functions of T_k), and irradiance conditions represented by Iph_k and Isc_{k+1}.

$$Vo_{k,k+1} = -B_k \cdot W_0\left(A_k \cdot c_2 \cdot \exp\left(c_2 \cdot \left(I_{Tk} - Isc_{k+1}\right)\right)\right) - Isc_{K+1} \cdot \left(Rs_k + Rh_k\right) + Rh_k \cdot I_{Tk} \tag{11}$$

Nevertheless, Equation (11) only concerns two modules. In a string with multiple modules, the minimum string voltage in which the *k-th* bypass diode is turned on, can be calculated by the sum of all inflection voltages contributions of the modules with higher PV currents (modules from *1* to *k-1*) as given in Equation (12) [5], where $k \circ [2,n]$ and $Vo_{m,k}$ is the voltage contributed by the *m-th* module to the *inflection voltage* of *k-th* module.

$$Vo_k = \sum_{m=1}^{k-1} Vo_{m,k} \tag{12}$$

It is important to remark that a string with n modules can exhibit *n-1* inflection voltages, as maximum, which means that each module of the string is subjected to a different operation condition due to its parameter tolerances and/or atmospheric conditions (radiation and temperature). For example, Figure 3 presents the current vs. voltage (I vs. V) curves of four ERDM85 modules under different irradiance conditions with its corresponding inflection voltages calculated by using Equation (11). Figure 4 illustrates the three inflection voltages of a string composed by the four modules; those points were calculated by using Equation (12).
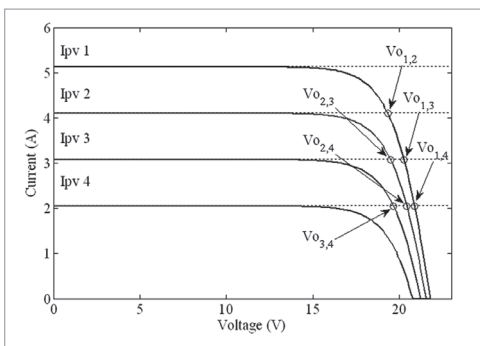

Figure 3. Inflection voltages on I vs. V curves of PV modules 1 to 4 under irradiance of 1, 0.8, 0.6 and 0.4 kW/m² and a cell temperature of 25 °C.
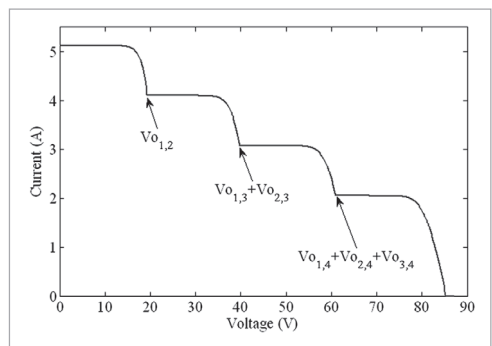

Figure 4. Inflection voltages on I vs. V curve a string with 4 PV modules under irradiance of 1, 0.8, 0.6 and 0.4 kW/m² and a cell temperature of 25 °C.

4. String current calculation using inflection voltages

The proposed model calculates the string voltage as the sum of the modules and blocking diode voltages. The blocking diode voltage is given by Equation (13), where V_{bd} and I_{bd} are the voltage and current of the blocking diode, respectively, A_{bd} is the reverse saturation current, a is the ideality factor that depends on the semiconductor material, and k, T and q were defined at the beginning of Section 2.

$$V_{bd} = \left(\frac{a \cdot k \cdot T}{q} \cdot \ln\left(\frac{I_{bd} + A_{bd}}{A_{bd}} \right) \right) \tag{13}$$

Since the modules and blocking diode voltages are defined in function of the string current, as given in Equations (9) and (13), the string current can be calculated as Equation (14), where j represents the number of the string, ac is the number of active modules in the string, and I_j represents the current of the j-th string.

$$Vstr_j = V_{1,j}(I_j) + V_{2,j}(I_j) + \dots + V_{ac,j}(I_j) - Vbd_j(I_j) \tag{14}$$

Notice that it is possible to construct the I vs. V curve for the j-th string by applying a numerical method for solving I_j in Equation (14), for each string voltage ($Vstr_j$) in the range from 0 to $n'''Voc_{STC}$ V (n is the number of modules in the string) with fixed steps (dV).

Another important aspect of the model concerns the number of terms of Equation (14), which depends on the number of active PV modules. For example, in a string with five modules, where two bypass diodes are active and three inactive, then Equation (14) will have four terms in its right hand side, where three terms correspond to the non-bypassed modules and one term corresponds to the blocking diode voltage. The number of non-bypassed modules is calculated from the string voltage and the inflection voltages: if $Vstr_j$ is higher than Vo_k, then k modules are active, otherwise if $Vstr_j$ is lower than Vo_k but higher than Vo_{k-1}, then k-1 modules are active.

The inflection voltages also allow restricting the possible range of the I_j solution. For example, if the string voltage is greater than Vo_{k-1} and lower than Vo_k, then the solution of I_j is between $Isc_{k-1,j}$ and $Isc_{k,j}$; therefore the numerical method used to solve the Equation (14) has a restricted range for the solution search and the starting point can be set close to the solution. Such restrictions yield a reduction in the convergence time of the numerical method compared to previous models [4][5] that do not provide a bounded search range or starting points for the solutions of the non-linear equation systems.

5. PV field evaluation

As described in Section 3, the proposed model considers a PV field composed by m strings of n modules connected in parallel (Figure 2). Therefore, to evaluate a PV field the analysis of Section 4 must be performed for each string. The final I vs. V curves of the whole PV field are then obtained by adding the currents of each module in each string voltage from 0 V to Voc_{sys} V ($Voc_{sys} = n.Voc_{STC}$). The complete procedure is presented in the flow chart of Figure 5. Such an algorithm was implemented in Matlab for performing analyses and simulations of the PV field as well as to obtain electrical characteristics under different mismatching conditions.
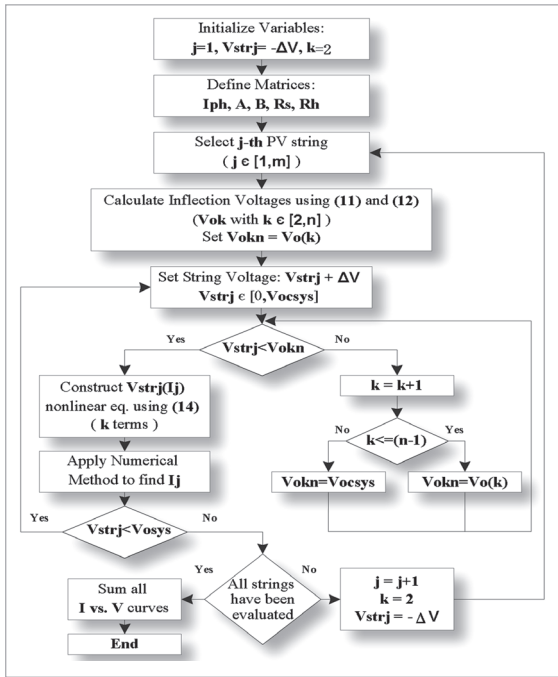
Figure 5. Flow chart of the proposed model.

6. Model parameters extraction

Without loss of generality, three series-connected PV panels (PV1, PV2 and PV3), with different temperature and irradiance conditions are considered in order to illustrate the procedure used to obtain the model parameters. I vs. V and P vs. V curves of the system are presented in Figure 6. In such a figure, the curves are divided into three sectors; sector 1 goes from 0 V to the first inflection voltage (Vo_2), sector 2 from Vo_2 to the second inflection voltage (Vo_3) and sector 3 from Vo_3 to the open circuit voltage of the system (Vo_{sys}). In the first sector, only the module with highest irradiance is delivering power since the string voltage is smaller than Vo_2 and Vo_3; hence the bypass diodes of the other two modules are active. In sector two there are two modules delivering power because $Vstr$ is between Vo_2 and Vo_3. In sector three, all bypass diodes are inactive because $Vstr$ is grater then Vo_2 and Vo_3, so the three PV modules are delivering power.

The first step in the parameter extraction is to find the inflection points dividing the three sectors; in this way current values for 0V, Vo_2, Vo_3 and string open circuit voltage (Vo_{sys}) are identified. The current for 0 V and Vo_{sys} are short circuit current and 0 A, respectively. To identify the current values for inflection voltages it is possible to scan the I vs. V curves and detecting the changes in dI/dV from a large negative value to a value near to cero. The first step delivers the four points illustrated in Table I and depicted in Figure6.

The second step is to enter the voltage and current data of PV1, which correspond to the values from P1 to P2 (sector 1) in the Matlab curve fitting tool box, which search the best values for parameters A_1, B_1, Rs_1 and Rh_1 (Table II) to adjust Equation (10) with the experimental data for a given value of Iph_1 (Iph_1=3.68 A for sector 1, see Table II).

Table I. Sector Division Points

Point	Current (A)	Voltage (V)
P1	3.68	0
P2	2.56	11.18
P3	0.41	34.99
P4	0	55.99

Table II. Model parameters of 3 PV panel

ModelParameter	PV 1	PV 2	PV 3
Iph (A)	3.68	2.56	0.41
A (µA)	10	0.006	7e-4
B	1.143	1.076	0.9968
Rs (Ù)	0.990	0.936	0.0286
Rh(Ù)	104.04	55.99	1752.4

Step three is to calculate the voltage of the PV2. As mentioned before, in sector 2 there are 2 PV modules active and 1 bypassed, therefore the voltage for the module 2 is given by Equation (15); where V_1 is the output voltage of PV1, V_2 is the output voltages of PV2 and $Vstr$ is the string voltage.

$$V_2 = Vstr - V_1 \qquad (15)$$

With the parameters of the PV1 module it is possible to predict its voltage from the currents values of the

sector 2 (from P2 to P3) by using Equation (9), therefore $Vstr$ and V_1 from P2 to P3 are known and V_2 can be calculated for each current value of sector 2. Taking in account the current for all PV modules is the same, the step four is to load the voltage and current values of PV2 in the curve fitting tool box for finding A_2, B_2, Rs_2 and Rh_2 by using Equation (10) with Iph_2=2.56 A (Table II).

Now continuing with step 4, in sector 3 (from P3 to P4) the three PV modules are active; hence the PV3 voltage is given by Equation (16).

Figure 6.Characteristics for one string of three panels ERDM85. (a) I vs. V curve. (b) P vs. V curve.

$$V_3 = Vstr - V_1 - V_2 \tag{16}$$

Similarly to step three, in step fifth, the voltage of PV1 and PV2 are predicted from each current value of sector 3 using Equation (10) because their parameters and photovoltaic currents are known. Step sixth is to load voltage and current data of PV3 in the curve fitting tool box for finding A_3, B_3, Rs_3 and Rh_3 with Iph_3=0.41 A (Table II) that match Equation (10) with the electrical data of PV3.

Moreover, although the example used to explain this procedure considers all PV modules have different conditions, it is also possible to use it when two or more equal modules have the same irradiance and temperature with a small modification in the procedure explained before.

Considering two consecutive points that delimits the sector k (Pk and Pk+1). Using Equation (17) it is possible to find the number of equal modules that share temperature and irradiance conditions in that sector. Here V_k and V_{k+1} are the starting and end points of the sector k, Voc_{STC} is the open circuit voltage of one PV module, and *ceil(a)* rounds a number to the nearest integer greater than *a*.

$$Eq_{PVk} = ceil\left(\frac{V_k - V_{k+1}}{Voc_{sys}}\right) \tag{17}$$

Once Eq_{PVk} is calculated, the voltage and current values of section k are introduced into the curve fitting tool box as explained before in this section; however the equation used for fitting with the experimental data for finding the parameters of one of the Eq_{PVk} modules (A_k, B_k, Rs_k and Rh_k for a given Iph_k) is shown in Equation (18), where *á* is defined in Equation (19). Note that the Equation (18) is the same relation given in Equation (10) but replacing $V_{i,j}$ by $V_{i,j}/Eq_{PVk}$ that corresponds to the voltages of one module in the sector k.

$$I_j = -\frac{B}{Rs} \cdot Wo(\alpha) + \frac{Iph_{i,j} + A - \dfrac{V_{i,j}}{(Rh \cdot EqPVk)}}{1 + \dfrac{Rs}{Rh}} \tag{18}$$

$$\alpha = \frac{A \cdot Rs}{B \cdot \left(1 + \dfrac{Rs}{Rh}\right)} \exp\left(\frac{\dfrac{V_{i,j}}{EqPVk} + Rs \cdot (A + Iph_{i,j})}{B \cdot \left(1 + \dfrac{Rs}{Rh}\right)}\right) \tag{19}$$
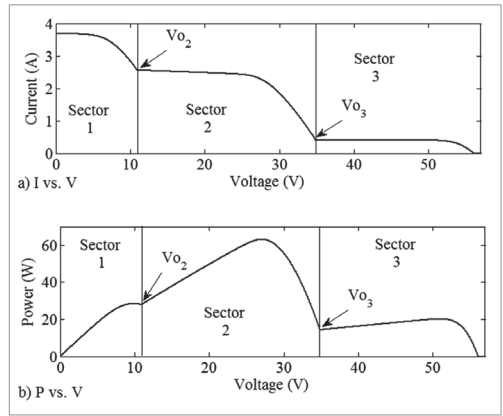
7. Model validation

To validate the proposed model, one experiment was carried out using two ERDM85 PV panels connected in series, a connection table, and an electronic load (BK 8514) connected to a PC, as depicted in Figure 7. In addition, a Matlab script was used to perform voltage sweeps on the PV string and to record the PV current and voltage from the DC load sensors.

Moreover, since the experiment was performed in a single string, the blocking diode was neglected, however, when the PV field has multiple strings all blocking diodes have to be considered. The ERDM85 PV panels have only one bypass diode and its electrical parameters are given in Table III [15].
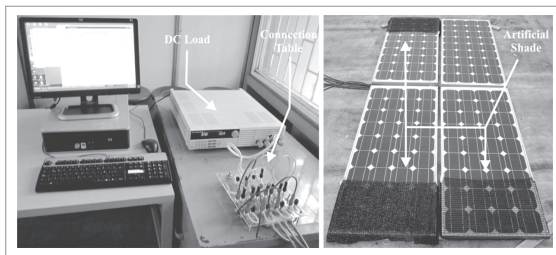

Figure 7. Experimental platform.

Table III. Datasheet Parameters of ERMD85

Parameter	Value	Unit
Open Circuit Voltage (Voc$_{STC}$)	21.78	µA
Short Circuit Current (Isc$_{STC}$)	5.13	A
MPP Voltage (Vmpp)	17.95	V
MPP Current (Impp)	4.8	A
Voc Temperature Constant (KVoc)	-0.07405	V/ C
Isc Temperature Constant (KIsc)	0.0013	A/ C
Number of Cells (Ns)	36	

To provide a comparison with another model used to predict the PV array electrical behavior, the much simpler and widely accepted single-diode model neglecting both series and parallel resistors[5] was also simulated. To process such a simpler model, Equations (9) and (10) were replaced by Equations (20) and (21), respectively.

$$V_{i,j} = B \cdot \ln\left(\frac{Iph_{i,j} - I_j + A}{A} \right) \tag{20}$$

$$I_j = Iph_{i,j} - A \cdot \exp\left(\frac{V_{i,j}}{B} \right) \tag{21}$$

In the experiment, both PV panels were partially covered with a plastic net to reproduce the partial shading effects (Figure 7). Table IV presents the model parameters for the two modules calculated by following the methodology described in Section 6. Figure 8 shows experimental and model predicted data in discrete and continuous line, respectively. The voltage sweep was performed from 0.1 V to 37.5 V with steps of 0.3 V and time intervals of 50 ms.

Concordance between the experimental and model predicted data is evident in Figure 8, even in the inflection voltages. On the other hand, relative errors of the proposed model are presented in Figure 9 (continuous line) along with those of the simplified model were Rs and Rh resistors were neglected (dashed line). In this figure, the current and power relative mean error of the proposed model were 1.01 % (0.01 A) and 1.01 % (0.34 W), respectively. The relative mean errors of the simplified model were 2.30 % (0.04 A) for the current and 2.30 % (0.74 W) for the power, which shows the accuracy improvement of the proposed model in a 2 PV panel string.

Table IV. Model parameters of 2 PV panel

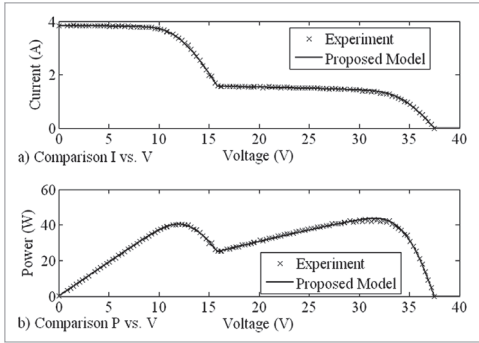| Model Parameter	PV 1	PV 2
Iph (A)	3.84	1.57
A (µA)	2.369	0.008
B	1.261	1.027
Rs (Ù)	1.0	0.202

Figure 8. Experimental ("x") and proposed model (line) characteristics for one string of two panels ERDM85. (a) I vs V curve. (b) P vs V curve.
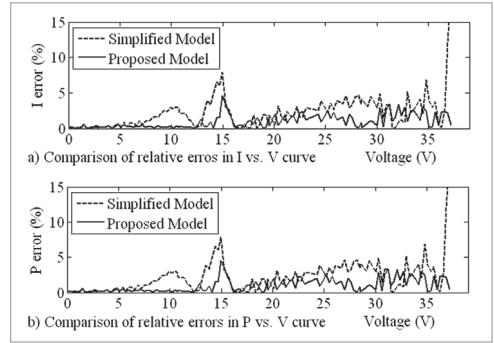

Figure 9. Relative errors considering Rs and Rh (continuous line) and neglecting them (dashed line) characteristic for a 2 PV string. . (a) I vs. V. (b) P vs. V.

8. Conclusions

This paper describes an algorithm to calculate the current and power of PV fields in non-uniform (mismatched) conditions and presents a procedure to obtain the parameters of the PV modules from the experimental measurements of the string current and voltages. The proposed model takes into account the series and parallel resistors of each PV module, considers the Shockley representation of the blocking diode, and approximates the bypass diode as an ideal switch. The equations of the circuital representation of the PV module were manipulated by applying the Lambert-W function to obtain the voltage of the PV module as an explicit function of the current. Such an expression allows designing a single equation per string by equalizing the voltages of all active modules and the blocking diode with string voltage. Then the field current is calculated by adding the currents of all strings with the same voltage.

It is worthy of attention that the definition of one equation for each string instead of a system of non-linear equations, as in[4] [5], reduces the computational costs since the calculations are restricted to the active PV modules. Moreover, the consideration of the parasitic series and parallel resistors of the model improves the accuracy of the predicted data with respect to the simplified single diode model. Therefore, the proposed model provides a tradeoff between accuracy and computational cost, and it is useful for testing MPPT systems and energetic evaluations.

The procedure devised for calculating the parameters of the PV modules divides the current vs. voltage curve of each string into sections (zones between two consecutive inflection points) and calculates the number of active modules per section, then it fits the current equation of the PV modules with the experimental data for obtaining the experimental parameters values. Using this procedure it is not required to scan the I vs. V curves of each PV panel separately to calculate its parameters; besides, the parameterization of the PV modules could be performed periodically for the maintenance of the PV array or to adjust the MPPT techniques.

The accuracy of the proposed model and the parameter calculation procedure were experimentally tested and compared with a benchmark. The results show that the relative mean error in the prediction of the PV power can be reduced by almost 50% using the adopted complete PV model. Further experiments could be performed for validating the proposed model for other PV technologies like polycrystalline cells, thin film cells or organic cells. For such technologies the parameters values may vary but the parameterization and modeling procedures are the same.

The algorithm presented in Section 5 is simple to implement in a standard programming language like C or Matlab for estimating the power of a PV field under different mismatching profiles and different array configurations. Such information could be used to define the best distribution of the PV panels in urban applications, which generally have fixed surrounding objects that produce predictable shades. Moreover, the proposed model could be used to estimate the energy production of a PV array, which is useful in the design of PV plants and in the calculation of the return of investment time.

Although the proposed model applies for PV arrays in Series-Parallel configuration (SP), there are other PV array configurations for interconnecting PV modules like Honey Comb (HC), Bridge-Linked (BL), and Total Cross-Tied (TCT). The latter is able to mitigate the mismatching effects in a PV array, thus it has been widely used in the PV array reconfiguration. Future work may consider extending the methodology applied in this paper to PV arrays in TCT configuration in order to perform comparisons with the SP configuration and to determine which is able to deliver the higher power for a given mismatching condition.

References

[1]   R. Faranda, S. Leva, and V. Maugeri, "MPPT techniques for PV Systems: Energetic and cost comparison",*2008 IEEE Power and Energy Society General Meeting - Conversion and Delivery of Electrical Energy in the 21st Century*, 2008, pp. 1-6.

[2]   IEA, *"Technology Roadmap Solar photovoltaic energy"*, 2010.

[3]   IEA, *"Photovoltaic Power Systems Programme, Annual Report 2010"*, 2010.

[4]   Giovanni Petrone, Giovanni Spagnuolo, and MasimoVitelli, "Analytical model of mismatched photovoltaic fields by means of Lambert W-function". *Solar Energy Materials and Solar Cells*, vol. 91, no. 18, Nov. 2007, pp. 1652-1657.

[5]   Giovanni Petrone and Carlos Ramos, "Modeling of photovoltaic fields in mismatched conditions for energy yield evaluations". *Electric Power Systems Research*, vol. 81, no. 4, Apr. 2011, pp. 1003-1013.

[6]   Hiren Patel and Vivek Agarwal, "MATLAB-Based Modeling to Study the Effects of Partial Shading on PV Array Characteristics". *Energy Conversion, IEEE Transactions on*, vol. 23, no. 1, Mar. 2008, pp. 302-310.

[7]   H. Tsai,C. Tu and Y. Su, "Development of generalized photovoltaic model using MATLAB/SIMULINK",*Proceedings of the World Congress on Engineering and Computer Science 2008*, 2008, pp. 1-6.

[8]   Marcelo Villalva, Jonas Gazoli, and Ernesto Filho, "Comprehensive Approach to Modeling and Simulation of Photovoltaic Arrays". *IEEE Transactions on Power Electronics*, vol. 24, no. 5, May 2009, pp. 1198-1208.

[9]   Daniel Chan and Jacob Phang, "Analytical methods for the extraction of solar-cell single- and double-diode model parameters from I-V characteristics". *IEEE Transactions on Electron Devices*, vol. 34, no. 2, Feb. 1987, pp. 286-293.

[10] R. Corless, G. Gonnet, D. Hare, D. Jeffrey, and D. Knuth, "On the Lambert W function". *Advances in Computational Mathematics*, vol. 5, no. 1, Dec. 1996, pp. 329-359.

[11] W. Desoto, S. Klein, and W. Beckman, "Improvement and validation of a model for photovoltaic array performance". *Solar Energy*, vol. 80, no. 1, Jan. 2006, pp. 78-88.

[12] J.D. Bastidas, C. A. Ramos-Paja, G. Spagnuolo andG. Petrone, "Modeling of photovoltaic fields in mismatching conditions by means of inflection voltages", *2012 Workshop on Engineering Applications (WEA)*, 2012, pp. 1-6.

[13] Ramaprabha Ramabadran and Badrilal Mathur, "Matlab based modeling and performance study of series connected SPVA under partial shaded conditions". *Journal of Sustainable Development*, vol. 2, no. 3, Nov 2009, pp. 85-94.

[14] ERDM, *"Datasheet ERDM 75-85 W." Available in: http://www.erdmsolar.com/shop/images/stories/file/SM580.pdf*.

Juan David Bastidas Rodríguez

Ingeniero Electrónico de la Universidad del Valle de Cali, Colombia. Actualmente es estudiante de Doctorado en Ingeniería en el campo de control de sistemas fotovoltaicos in la Universidad del Valle y pertenece al Grupo de Investigación en Control Industrial (GICI) de la Universidad del Valle y del Grupo de Investigación en Automática (GAUNAL) de la Universidad Nacional de Colombia.

Carlos Andrés Ramos-Paja

Ingeniero Electrónico y Mágister en Automática en la Universidad Valle, Colombia; tiene título de Maestría en Ingeniería Electrónica de la Universitat Rovira i Virgili de Tarragona, España. PhD Universitat Rovira i Virgili de Tarragona, España.Actualmente se desempeña como profesor en el área de Desarrollo Tecnológico e Industrial en la Universidad Nacional de Colombia, sede Medellín, y pertenece al grupo de investigación GAUNAL donde realiza estudios sobre electrónica de potencia, fuentes alternas de energía y control de generadores sincrónicos.

Edinson Franco Mejía

Ingeniero Electricista y Mágister en Automática en la Universidad del Valle de Cali, Colombia.PhD en la Universidad del Valle, de Cali, Colombia. Actualmente se desempeña como profesor titular en el área de Automática y Control y Director del programa de Posgrados de Ingeniería Eléctrica y Electrónica en la Universidad del Valle. Pertenece al Grupo de Investigación en Control Industrial (GICI).