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On the fuzzy extension principle for LP problems with Interval Type-2 Technological Coefficients

Acerca del principio de extensión para problemas LP con parámetros difusos Tipo-2 de Intervalo

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Abstract

A special kind of Linear Programming (LP) problems involve linguistic uncertainty that can be represented by Interval Type-2 Fuzzy numbers using the extension principle for fuzzy sets. This principle has been widely used in decision making, computation of a function of fuzzy sets, fuzzy optimization, etc, so this paper focuses to clarify some aspects about its use in LPs with interval Type-2 fuzzy technological parameters.

Key words: Fuzzy Sets, Linear Programming, Extension Principle.

Resumen

Una familia especial de problemas de programación lineal (LP) incluyen incertidumbre lingüística que puede ser representada a través de números difusos Tipo-2 de intervalo que a su vez se operacionalizan a través del principio de extensión. Este principio ha sido ampliamente utilizado en toma de decisiones, cómputo de funciones de conjuntos difusos, optimización difusa, etc, por lo que en este artículo hacemos algunas claridades acerca de su aplicación en problemas LP con coeficientes tecnológicos definidos como números difusos Tipo-2 de intervalo.

Palabras claves: Conjuntos Difusos, Programación Lineal, Principio de Extensión.

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1. Introduction

In some situations decision making is performed by a group of people instead of a single person, so disagreement, ambiguity, and other uncertainty sources can affect the selection of a particular parameter. The main idea of combining Linear Programming (LP) models to fuzzy sets is to include the opinion of multiple experts who define the left hand side parameters on an LP problem, namely a_{ij}.

In some practical applications, the parameters a_{ij} are just realizations of an unknown random process that can be represented through multiple experts perceptions, so the analyst has to find a solution given those operation conditions. This way, there is a need for correlate practical issues (random realizations of a_{ij}) to experts perceptions and opinions about a_{ij}.

On the other hand, there is a need for computing the membership degree that a random selection of technological coefficients of an LP problem has, to later analyze their results to what it is expected by experts.

Different approaches to Fuzzy Linear Programming (FLP) problems have been presented in bibliography. Rommelfanger [19], [20], [17], [18], Ramík [14], [15], Ramík & Řimánek [16], and Gasimov & Yenilmez [9] who treated the field of classical fuzzy sets and its application in LP problems.

Figueroa-García [4] has proposed a model for FLP with uncertain technological parameters defined as Interval Type-2 Fuzzy Numbers (IT2FN), Figueroa-García & Hernández [7], [6] proposed a method for solving LP problems with Type-2 fuzzy constraints, Figueroa-García, Chalco-Cano & Román-Flores [5], and Figueroa-García & Hernández [8] provide some definitions on fuzzy constraints and fuzzy ordering. Building upon these results, in this paper we apply them to the particular issue of handling random values of a_{ij} in order to compute its optimal solution and it satisfaction degree.

This paper focuses on how to compute the membership degree of a random realization of a_{ij} given previous experts perceptions and/or opinions about a_{ij} which are represented using interval Type-2 fuzzy sets, and applied in LP problems. The paper is divided into six sections; a first one that introduces the main problem; a second one that shows some basic concepts on interval Type-2 fuzzy sets; a third one that presents the interval Type-2 fuzzy linear programming model; a fourth section introduces the extension principle for fuzzy sets; a fifth section presents and application example, and finally some concluding remarks are shown in section 6.

2. Basics on Type-2 fuzzy numbers

Firstly, we want to clarify some notations used throughout this paper. An Interval Type-2 Fuzzy Set (IT2FS) is denoted by emphasized capital letters $\tilde{A}$ whose membership function $\mu_{\tilde{A}}(x)$ is defined over $x \in X$. A classical fuzzy set μ_A is a measure of the affinity of a particular value $x \in X$ regarding a concept/word/label A, and $\mu_{\tilde{A}}(x)$ measures uncertainty of a value $x \in X$ regarding A.

In this paper, we use notations and definitions provided by Jerry Mendel [12]. A Type-2 fuzzy set is an ordered pair $\{((x,u),\mu_{\tilde{A}}(x,u)) : x \in X, u \in J_x \subseteq [0,1]\}$, where A is its linguistic label, so $\tilde{A}$ represents uncertainty around the word A; $\mathcal{F}_1(X)$ is the class of all Type-1 fuzzy sets over X; $\mathcal{F}_2(X)$ is the class of all Type-2 fuzzy sets over X; $\mathcal{P}(X)$ is the class of all crisp sets over X. A Type-2 fuzzy set can be represented as follows:

$$\tilde{A} : X \to \mathrm{F}([0,1])$$
$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) \mid x \in X\}$$
$$\tilde{A} = \{((x,u), \mu_{\tilde{A}}(x,u) \mid \forall x \in X, \forall u \in J_x \subseteq [0,1]\}$$

where J_x is the set of primary membership degrees of $x \in X$, $0 \leqslant \mu_{\tilde{A}}(x,u) \leqslant 1$, and u is its domain of uncertainty.

An IT2FS $\tilde{A}$ encloses an infinite amount of clasical fuzzy sets into its *Footprint of Uncertainty* (FOU), and A_e is a Type-1 fuzzy set embedded in its FOU (see Mendel et al [13]):

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) \,|\, x \in X\}$$

where $\mu_{\tilde{A}}(x)$ is completely characterized by $J_x \subseteq [0,1]$.

A simpler Type-2 fuzzy set is called *Interval Type-2 fuzzy set (IT2FS)* in which $u = 1$, and is characterized by two primary membership functions: Upper Membership Function (UMF) namely $\overline{\mu}_{\tilde{A}}$, and Lower Membership Function (LMF) namely $\underline{\mu}_{\tilde{A}}$. Figure 1 shows the set $\tilde{A}$.

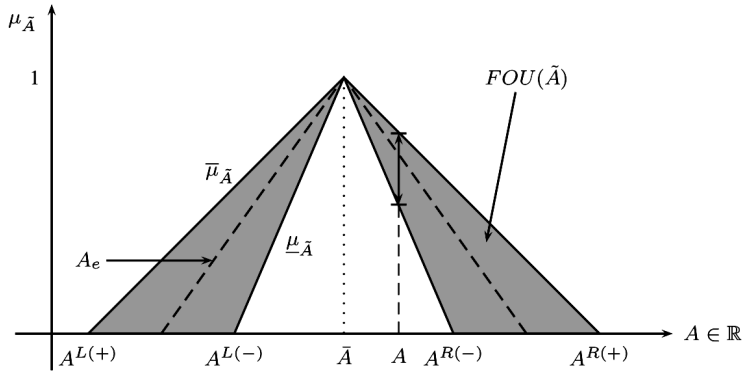

Figura 1. Interval Type-2 Fuzzy set $\tilde{A}$

As discussed by Figueroa-García, Chalco-Cano & Román-Flores [5], a Type-2 fuzzy number *(T2FN)* is considered as the extension of a Type-1 fuzzy number. This is, $\tilde{A}$ is a T2FS whose UMF and LMF are fuzzy numbers e.g a normal fuzzy set, ${}^{\alpha}A$ is closed interval for all $\alpha \in [0,1]$, and its support *supp*$(A) \in \mathcal{P}(\mathbb{R})$ is bounded which means that a fuzzy number is a convex fuzzy set as well.

The α-cut of A is defined as ${}^{\alpha}A = \{x \,|\, \mu_A(x) \geqslant \alpha\}$. This makes the computation of any function $f(x)$ of fuzzy sets easier. Here, $z^* = c'x^*$ comes from $f(\tilde{A}_i)(z^*)$, so we want to compute $f(\tilde{A}_i)(z^*)$ by using α-cuts instead of mapping x_i. The α-cut of A is simply the cut ${}^{\alpha}J_x = \{x \,|\, \mu_A(x) \geqslant \alpha\}$ done over its primary membership function J_x (see Figueroa [3],

and Figueroa-García, Chalco-Cano & Román-Flores [5]). A graphical representation of ${}^{\alpha}\tilde{A}$ is provided in Figure 2.

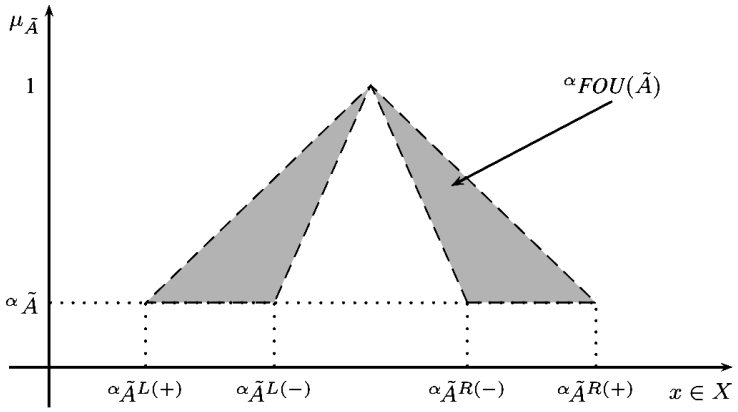

Figura 2. ${}^{\alpha}\tilde{A}$ of an Interval Type-2 Fuzzy set $\tilde{A}$.

3. The Interval Type-2 fuzzy LP problem

First, the classical crisp LP problem is basically is a model that relates a set of m inequalities whose goal is to maximize the value of an objective (a.k.a goal) function, as follows:

$$\begin{aligned} \underset{x}{\text{Max}}\ z &= c'x + c_0 \\ s.t. & \\ Ax &\leqslant b \\ x &\geqslant 0 \end{aligned} \tag{1}$$

where $x \in \mathbb{R}^n$, $c \in \mathbb{R}^n$, $c_0 \in \mathbb{R}$, $A \in \mathbb{R}^{n \times m}$, and $b \in \mathbb{R}^m$.

Now, when technological parameters A cannot be well defined or they are bad measured, we need to use other methods to get those parameters. A popular way to get them is by using experts opinions, so the use of fuzzy sets to quantify its perceptions about A become an important information source. Commonly in the industry, there are more than one expert who can provide valuable information about A, so we need to comprise all their perceptions and opinions into a single measure: an IT2FN.

This leads us to re-define the crisp LP model as an IT2FLP, as follows:

$$\begin{aligned} \underset{x}{\text{Max}}\ z &= c'x + c_0 \\ s.t. & \\ \tilde{A}x &\preccurlyeq b \\ x &\geqslant 0 \end{aligned} \tag{2}$$

where $x \in \mathbb{R}^n$, $c \in \mathbb{R}^n$, $c_0 \in \mathbb{R}$, $\tilde{A} \in \mathcal{F}_2(X)$.

This model cannot be solved in a closed form, so the idea in this paper is to show how to compute the membership degree of any crisp solution and how to understand its sense. Based on the results of Figueroa-García [4] we do a complement of his results by clarifying some concepts about random solutions instead of α-cuts (which is also a reasonable method).

The α-cuts approach proposed by Figueroa-García [4] is way to go from $\tilde{A}$ to $\tilde{z}$ (which is the IT2FS of optimal solutions), but it does not include random choices of A. A random choice of A is a selection of different a_{ij} that depends on the current conditions of the system, and mostly are just available conditions under the system has to operate, so the analyst has no any guarantee that their results are the best possible or even good, they are just solutions of the IT2FLP.

Next section is dedicated on explain how to compute the membership degree that any random solution has, and how to understand its meaning.

4. Extension principle for IT2FS

A first approach to find an appropriate way for modeling fuzzy functions is given by the Zadeh's *Extension principle* (see Bellman & Zadeh [1], Klir & Yuan [10], and Mendel [12]). Let f be a function such as $f : X_1, X_2, \cdots, X_n \rightarrow z$, and A_i is a fuzzy set in X_i, $i = 1, 2, \cdots, n$ with $x_i \in X_i$, then we have

$$f(A_1, A_2, \cdots, A_n)(z) = \sup_{z=f(x_1,x_2,\cdots,x_n)} \min_i [A_1(x_1), A_2(x_2), \cdots, A_n(x_n)] \tag{3}$$

The fuzzy extension principle projects any function e.g $z = f(x_1, x_2, \cdots, x_n)$ to a fuzzy set by using their memberships $A_1(x_1), A_2(x_2), \cdots, A_n(x_n)$. Its extended version is as follows.

Definition 1 *Let $A_x \in supp(\tilde{A})$ be a matrix of crisp technological coefficients, $A_x \in \mathbb{R}^{m,n}$, and $z_x^* = c'x^*$ be the optimal solution of the LP for A_x. Then we have*

$$f(A_x)(z_x^*) = \min_{i' \in k} \{[\underline{\mu}_{\tilde{A}_{1'}}, \overline{\mu}_{\tilde{A}_{1'}}], \cdots, [\underline{\mu}_{\tilde{A}_{i'}}, \overline{\mu}_{\tilde{A}_{i'}}], \cdots, [\mu_{\underline{A}_{k'}}, \mu_{\overline{A}_{k'}}]\}, \tag{4}$$

$$\underline{\mu}_{\tilde{A}_{i'}} = \min_j \{ \underline{\mu}_{\tilde{A}_{i'1}}, \cdots, \underline{\mu}_{\tilde{A}_{i'j}}, \cdots, \underline{\mu}_{\tilde{A}_{i'n}} \}, \tag{5}$$

$$\overline{\mu}_{\tilde{A}_{i'}} = \min_j \{ \overline{\mu}_{\tilde{A}_{i'1}}, \cdots, \overline{\mu}_{\tilde{A}_{i'j}}, \cdots, \overline{\mu}_{\tilde{A}_{i'n}} \}. \tag{6}$$

where $i' \in k$ is the set of all binding constraints of the optimal program.

This Definition computes the membership degree of any random solution z_x^* regarding the set $\tilde{z}$. Basically it is computed using the membership degree that A_x has regarding the sets $\tilde{A}_{ij}$, that is $\overline{\mu}_{\tilde{A}_{ij}}$ and $\underline{\mu}_{\tilde{A}_{ij}}$ which finally lead to $f(A_x)(z_x^*)$.

5. Application example

To illustrate how an IT2FLP works, we present the following example. Suppose that a company has to plan the production quantity (in thousands) of two products x_1 and x_2, where every sold product returns $c_1 = 2$ and $c_2 = 3$ thousand US dollars (profits) per unit, and its production requires two materials which are available by $b_1 = 12$ and $b_2 = 15$ tons, respectively. The material consumption a_{ij} per product x_1, x_2 is uncertain since historical data is absent, so the analyst is encouraged to find a way to plan the best production quantities that maximize profits.

Given hypothetical normal operation conditions, it is supposed that the material consumption per product should be $a_{11} = 1, a_{12} = 4, a_{21} = 3$ and $a_{22} = 2$, but it is a hard supposition since we do not have historical information about the material consumption in order to verify the performance of the company. To have a better idea about the system, we have to ask the people involved into production planning, so we enquire to five experts (people on manufacturing, engineering, mechanical processes, etc) about their perception around material consumption of every product.

Now, both uncertain parameters and information that comes from multiple experts (perceptions and opinions) leads us to deal with Type-2 fuzzy uncertainty, so a_{ij} turns into $\tilde{A}_{ij}$. This way, the experts are encouraged to provide its opinions about *"Normal material consumption"* of every $\tilde{A}_{ij}$ using two boundaries: *pessimistic* and *optimistic*. The description of the IT2FLP is given next:

$$\begin{aligned} \underset{x_j}{\text{Max}} \;\; & z = 2x_1 + 3x_2 \\ & s.t. \\ & \tilde{1}_{11}x_1 + \tilde{4}_{12}x_2 \precsim 12 \qquad (7) \\ & \tilde{3}_{21}x_1 + \tilde{2}_{22}x_2 \precsim 15 \qquad (8) \\ & x_j \geqslant 0 \end{aligned}$$

The analyst has to use the information provided by the experts, which usually is described using sentences such as *"I think that the parameter (i, j) should be between a and c"*, or *"I think that the most possible value of the parameter (i, j) should be b"*. Now, the five experts namely $E_1, \cdots, E_5$ were asked for the parameters a_{ij} using the above sentences having the same idea about its most possible value (provided as starting information in the model), but they are not agree about their perceptions around its boundaries.

In this paper, the UMF and LMF of every a_{ij} are triangular membership functions. A triangular membership function $T(a, b, c)$ is defined by an *optimistic* boundary a, an *expected* value b, and a *pessimistic* boundary c, as follows:

$$T(a,b,c) = \begin{cases} 0, & x \leqslant a \\ \dfrac{x-a}{b-a}, & a \leqslant x \leqslant b \\ 1, & x = b \\ \dfrac{c-x}{c-b}, & b \leqslant x \leqslant c \\ 0, & x \geqslant c \end{cases} \tag{9}$$

The complete description of $a_{ij}$is shown next:

$$\begin{array}{ll} \bar{\mu}_{\tilde{A}_{11}} = T(0,1,3) & \underline{\mu}_{\tilde{A}_{11}} = T(0.5,1,1.5) \\ \bar{\mu}_{\tilde{A}_{12}} = T(2,4,7) & \underline{\mu}_{\tilde{A}_{12}} = T(3,4,5.5) \\ \bar{\mu}_{\tilde{A}_{21}} = T(1,3,5) & \underline{\mu}_{\tilde{A}_{21}} = T(2,3,4) \\ \bar{\mu}_{\tilde{A}_{12}} = T(0,2,5) & \underline{\mu}_{\tilde{A}_{12}} = T(0.5,2,3.5) \end{array}$$

First, we solve the IT2FLP using the method proposed by Figueroa-García [4] which computes a predetermined amount of α-cuts (in this case 10), and then solve the 4α crisp problems using 1. This leads to solve a crisp LP for ${}^{\alpha}\tilde{A}^{L(+)}$,${}^{\alpha}\tilde{A}^{L(-)}$,${}^{\alpha}\tilde{A}^{R(-)}$ and ${}^{\alpha}\tilde{A}^{R(+)}$ (see Figure 2). Finally the set of optimal solutions $\tilde{z}$ is done by using the representation theorem and the extension principle for fuzzy sets (see Klir & Yuan [11]), this is $f(\tilde{A})(z^*)$. The set of optimal solutions $\tilde{z}$ is shown in Figure 3

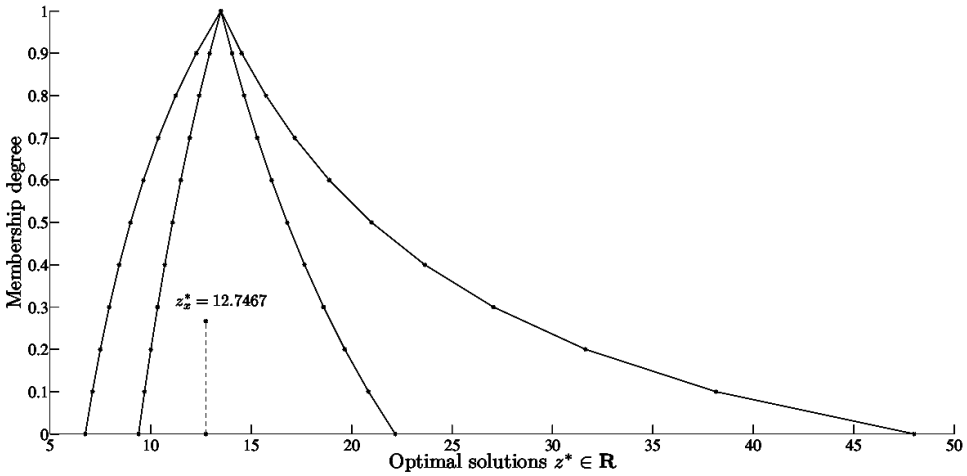

Figura 3. Set of optimal solutions $\tilde{z}$

Figure 3 show the whole set of optimal solutions of the IT2FLP as a function of α. In practice there is no any certainty that the values of A corresponding to ${}^{\alpha}\tilde{A}$ occur. Nay, what we can see in practical applications are *random realizations* of $\tilde{A}$ namely A_x, which indeed lead to an optimal solution z_x^* having a membership degree that can be computed by using Definition 1.

Figueroa-García [4] did not consider random realizations A_x of $\tilde{A}$ since that method maps uncertainty using α-cuts while this paper is focused to compute the membership of A_x regarding the set $\tilde{z}$ of optimal solutions proposed by Figueroa-García [4]. The current proposal complements the results of Figueroa-García [4] in the sense that he maps all uncertainty of $\tilde{A}$ while in this paper the membership of a random value A_x is computed and enclosed into $\tilde{z}$.

Now, suppose that the following values $A_x \rightarrow a_{11} = 0.7, a_{12} = 6.2, a_{21} = 3.1$ and $a_{22} = 1.4$ are observed, leading to $z_x^* = 12.7467$. The question is, how satisfactory is this solution to the experts?. This can be computed using Definition 1 as follows:

$$\overline{\mu}_{\tilde{A}_{11'}} = 0.7,\ \underline{\mu}_{\tilde{A}_{12'}} = 0.2667,\ \underline{\mu}_{\tilde{A}_{21'}} = 0.95,\ \underline{\mu}_{\tilde{A}_{22'}} = 0.3$$
$$\underline{\mu}_{\tilde{A}_{11'}} = 0.4,\ \underline{\mu}_{\tilde{A}_{12'}} = 0,\ \underline{\mu}_{\tilde{A}_{21'}} = 0.9,\ \underline{\mu}_{\tilde{A}_{22'}} = 0.4$$

Now, the membership value of $f(A_x)(z_x^* = 12.7467)$ is computed as follows

$$\overline{\mu}_{\tilde{A}_{1'}} \min_{1}\{0.7, 0.2667\} = 0.2667, \overline{\mu}_{\tilde{A}_{2'}} = \min_{2}\{0.95, 0.3\} = 0.3$$
$$\underline{\mu}_{\tilde{A}_{1'}} \min_{1}\{0.4, 0\} = 0, \underline{\mu}_{\tilde{A}_{2'}} \min_{2}\{0.9, 0.4\} = 0.4$$
$$f(A_x)(z_x^*) = \min_{1,2}\{[0, 0.2667], [0.4, 0.3]\} = [0, 0.2667]$$

This means that the optimal $z_x^* = 12.7467$ is satisfactory to all experts in the range $[0, 0.2667]$. On the other hand, this interval of satisfaction degrees is not as large as the corresponding α-cut is, which means that arbitrary choices of $A_x \in \tilde{A}$ are not always the most satisfactory solutions of the problem.

A fully satisfactory crisp solution of this example is $z^* = 13.5$ coming from $x_1^* = 3.6, x_2 = 2.1$. It can be compared to the solution provided by A_x which is $z_x^* = 12.7467$ coming from $x_1^* = 4.18, x_2^* = 1.46$ in the sense that A_x is a random realization of $\tilde{A}$ while the crisp solution needs more materials than the random problem except from first material for second product. Other issue is that the crisp problem is the expected value of the problem which is a less possible combination of parameters. The random problem prefers to produce more units of x_1 than the crisp one and reduces the quantities of x_2 to have a better usage of materials.

Both examples provide different solutions for different operation points of the system, so every optimal solution provides a different way to operate the system.

6. Concluding remarks

We have clarified the meaning of the extension principle applied to IT2FLPs when using random choices of a_{ij} which are enclosed into $\tilde{A}_{ij}$. This kind of solutions are enclosed into the set of optimal solutions $\tilde{z}$, but their membership degrees are smaller than the α-cuts solutions which in practice means that random selections of a_{ij} are less satisfactory to the experts of the system even when they are feasible and optimal.

A practical example that shows the applicability of our results is presented to illustrate how a random choice can be optimal in a crisp sense, but less satisfactory to experts expectations about the system's performance.

Based on the IT2FLP model proposed by Figueroa-García [4], [2], and some definitions about fuzzy constraints provided by Figueroa-García, Chalco-Cano & Román-Flores [5] we have extended the applicability of IT2FLPs in presence of random parameters a_{ij} which is a common issue seen on practical applications. Now, the analyst can see how satisfactory an optimal solution is, given any combination of parameters a_{ij}.

Next steps lead to general modeling for FLPs with fuzzy costs, technological coefficients, and constraints which are more complex problems. Also the use of generalized Type-2 fuzzy sets comes as a new way to represent uncertainty, so its potential use in optimization arises as a new field to explore.

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