Research paper

A Comparison Between the Centroid and the Yager Index Rank for Type Reduction of an Interval Type-2 Fuzzy Number

Comparación entre el Índice de Yager y el Centroide para Reducción de tipo de un Número Difuso Tipo-2 de Intervalo

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Abstract

Context: There is a need for ranking and defuzzification of Interval Type-2 fuzzy sets (IT2FS), in particular Interval Type-2 fuzzy numbers (IT2FN). To do so, we use the classical Yager Index Rank (YIR) for fuzzy sets to IT2FNs in order to find an alternative to the centroid of an IT2FN.

Method: We use a simulation strategy to compare the results of the centroid and the YIR of an IT2FN. This way, we simulate 1000 IT2FNs of the following three kinds: gaussian, triangular, and non symmetrical in order to compare their centroids and YIRs.

Results: After performing the simulations, we compute some statistics about its behavior such as the degree of subsethood, equality and the size of the Footprint of Uncertainty (FOU) of an IT2FN. A description of the obtained results shows that the YIR is less wide than centroid of an IT2FN.

Conclusions: In general, YIR is less complex to obtain than the centroid of an IT2FN, which is highly desirable in practical applications such as fuzzy decision making and control. Some other properties regarding its size and location are also discussed.

Keywords: Interval Type-2 Fuzzy numbers, Yager Index, Ranking.

Language: (english).

Resumen

Contexto: Hay una necesidad por defuzzificar y rankear Conjuntos Difusos Tipo-2 de Intervalo (IT2FS), en particular Números Difusos Tipo-2 de Intervalo (IT2FN). Para ello, usamos el Índice de Yager (YIR) para conjuntos difusos aplicado a IT2FNs con el fin de encontrar una alternativa al centroide de un IT2FN.

Método: Usamos una estrategia de simulación para comparar los resultados del centroide y del YIR de un IT2FN. Así pues, simulamos 1000 IT2FNs de cada uno de los siguientes tres tipos: gausianos, triangulares y asimétricos para comparar sus centroides y YIRs.

Resultados: Después de realizar las simulaciones, se calculan algunas estadísticas de su comportamiento como el grado de cobertura y de igualdad relativas del YIR respecto al centroide así como el tamaño de la Huella de Incertidumbre (FOU) de un IT2FN. La descripción de los resultados obtenidos muestra que el YIR es menos amplio que el centroide.

Conclusiones: En general, el YIR es menos complejo de obtener que el centroide de un IT2FN, lo cual es altamente deseable en aplicaciones prácticas como toma de decisiones y control. Otas propiedades relacionadas con su tamaño y ubicación también son diiscutidas.

Palabras clave: Números Difusos Tipo-2 de Intervalo, Índice de Yager, Ranking.

1. Introduction

The use of fuzzy sets in last decades have become an important tool to solve problems that involve non probabilistic uncertainty. One of the most important open problems when using fuzzy sets regards to the way to find a crisp measure that represents the behavior of the set. This process is known as *Type reduction*, and the case of an *Interval Type-2 Fuzzy Number (IT2FN)* is even more complex than classical fuzzy sets. Also the low availability of efficient Type reduction methods leads to analyze the properties of the most used ones, the *IASCO algorithm* (see Melgarejo [12]) which comes from the *Enhanced Karnik-Mendel algorithm (EKM)* (see Karnik & Mendel [22]) and the *Yager Index Rank (YIR)* for IT2FNs in this case.

Fuzzy sets are specially useful in engineering problems whose statistical information is unreliable or absent, and one of the possible ways to obtain information is via experts opinions and perceptions. Type-2 fuzzy sets cover uncertainty coming from multiple experts perceptions, and the way how they perceive the the problem, so its applicability in engineering is wide, specially in control and decision making problems where multiple experts are involved (see Hu et al [7], Kahraman et al [9], Melgarejo & Peña [13], and Mendel & Wu [19]).

Following the results presented in WEA 2015 (see Figueroa-García & Pachón-Neira [6]), this time we compare the centroid to Yager index for type reduction of IT2FNs in order to see their properties. To do so, we simulate 1000 IT2FNs of three shapes: triangular, gaussian, and non symmetric triangular membership functions and compute their centroids, Yager indexes, and other interesting measures to provide some useful information to readers who want to implement Type-2 fuzzy sets/systems.

The paper is organized into 6 sections; Section 1 introduces the main problem; Section 2 presents some basics about Type-2 fuzzy sets; Section 3 introduces the centroid and Yager index of an

IT2FN; Section 4 presents the methodology used for comparing both methods; In section 5, the results of the experiments are presented, and finally Section 6 presents some concluding remarks of the study.

2. Basics on Interval Type-2 fuzzy sets

In this paper, we do not make any distinction between definitions of IVFSs and Interval Type-2 fuzzy sets (IT2FSs) given by Mendel [15] since they are equivalent (see Mendel [17], Bustince [2], Bustince et al. [1], and Türksen [21]). A Type-2 fuzzy set is then:

$$\tilde{A} : X \rightarrow \mathrm{F}([0,1]) \tag{1}$$

$$\tilde{A} = \{((x,u), J_x, f_x(u)) \mid x \in X; u \in J_x \subseteq [0,1]\} \tag{2}$$

$$\tilde{A} = \int_{x\in X}\int_{u\in J_x} f_x(u)/(x,u),\ J_x \subseteq [0,1] \tag{3}$$

where $\tilde{A}$ represents uncertainty around the word A, J_x is the *primary membership* of x, u is its *domain of uncertainty*, and $\mathcal{F}_2(\mathbb{X})$ is the class of all Type-2 fuzzy sets (see Mendel [15], [19]).

This way, $\tilde{A}$ is composed by an infinite amount of embedded Type-1 fuzzy sets namely A_e. Every element x has associated a set of primary memberships J_x weighted by a *Secondary* fuzzy set $f_x(u)$ where u is the domain of uncertainty of x, $u \in J_x \subseteq [0,1]$. Now, an IT2FS is a simplification of a T2FS since its secondary membership function is assumed to be 1, as shown as follows.

$$\tilde{A} = \int_{x\in X}\int_{u\in J_x} 1/(x,u) = \int_{x\in X}\left[\int_{u\in J_x} 1/u\right]\Big/ x, \tag{4}$$

where x, u are the primary and secondary variables, and $f_x(u)/u = 1$ is the secondary membership function.

Uncertainty about the word A is conveyed by the union of all of J_x into the *Footprint Of Uncertainty* of $\tilde{A}$, namely FOU$(\tilde{A})$, which is bounded by two functions: An *Upper* membership function *UMF*$(\tilde{A}) = \bar{\mu}_{\tilde{A}}(x) \equiv A^U$ and a *Lower* membership function *LMF*$(\tilde{A}) = \underline{\mu}_{\tilde{A}}(x) \equiv A^L$. FOU$(\tilde{A})$ is shown in Figure 1.

In Figure 1, $\tilde{A}$ is an IT2FS, the universe of discourse for the primary variable x is the set $x \in X$, the *support* of $\tilde{A}$, *supp*$(\tilde{A})$ is the interval $x \in [\bar{\check{x}}, \bar{\hat{x}}]$ and $\mu_{\tilde{A}}$ is a triangular membership function with parameters $\bar{\check{x}}, \bar{\hat{x}}, \underline{\check{x}}, \underline{\hat{x}}$ and $\bar{x}$.

Definition 2.1 (α-cut of an IT2FS) *Figueroa-García, Chalco-Cano & Román-Flores [5] have defined the α-cut of an IT2FS as follows:*

$$^{\alpha}\tilde{A} = \left[[\check{A}^U_\alpha, \check{A}^L_\alpha], [\hat{A}^L_\alpha, \hat{A}^U_\alpha]\right]. \tag{5}$$

where $^{\alpha}A_e = \{x \mid \mu_{A_e}(x) \geqslant \alpha\}$ is an α-cut done over an embedded set $A_e \in \tilde{A}$. The symbol $\int$ denotes fuzzy union, so $^{\alpha}\tilde{A}$ is the union of all $^{\alpha}A_e$.

A graphical representation of ${}^{\alpha}\tilde{A}$ is provided in Figure 2.

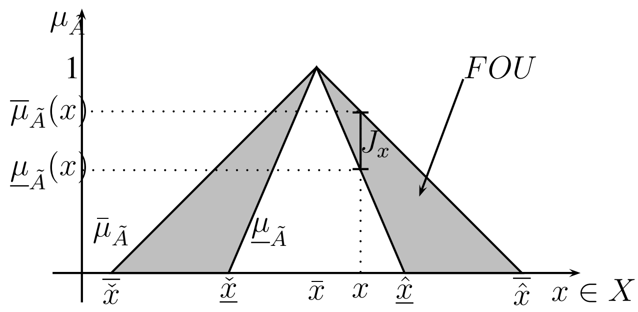

Figure 1. Interval Type-2 Fuzzy set $\tilde{A}$

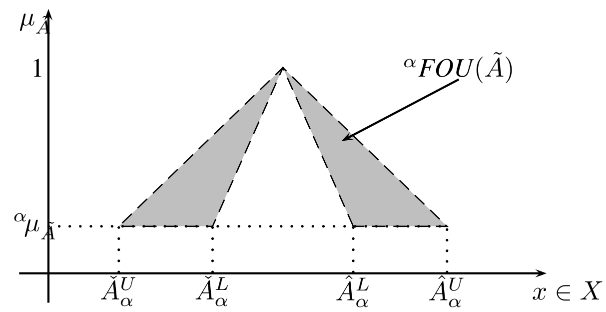

Figure 2. ${}^{\alpha}\tilde{A}$ of the set $\tilde{A}$

2.1. Type-2 fuzzy numbers

In this paper, a Type-2 fuzzy number *(T2FN)* is considered as the extension of a Type-1 fuzzy number. This means that $\tilde{A}$ is an T2FS whose UMF and LMF are fuzzy numbers (e.g. normal and convex fuzzy subsets of $\mathbb{R}$, Zadeh [25]). ${}^{\alpha}A$ is closed interval for all $\alpha \in [0,1]$, and its support *supp*(A) is defined over $\mathbb{R}$. This also means that a fuzzy number is a normal and convex fuzzy set, as shown as follows.

Definition 2.2 (Type-2 Fuzzy Number) *Let* $\tilde{A} \in \mathcal{F}_2(\mathbb{R})$. *Then,* $\tilde{A}$ *is a Type-2 Fuzzy Number (T2FN) iff there exists a closed interval* $[a,b] \neq \emptyset$ *for each UMF* $(\bar{\mu}_{\tilde{A}})$ *and LMF* $(\underline{\mu}_{\tilde{A}})$ *such that*

$$\mu_{\tilde{A}}(x,u) = \begin{cases} 1 & \text{for } x \in [a,b], & u \in J_x \subseteq [0,1] \\ l(x,u) & \text{for } x \in [-\infty,a], & u \in J_x \subseteq [0,1] \\ r(x,u) & \text{for } x \in [b,\infty], & u \in J_x \subseteq [0,1] \end{cases} \tag{6}$$

where $l : (-\infty,a) \to \mathrm{F}([0,1]), u \in J_x \subseteq [0,1]$ *is monotonic non-decreasing, continuous from the right, and* $l(x,u) = 0$ *for* $x < \omega_1$, *and* $r : (b,\infty) \to \mathrm{F}([0,1]), u \in J_x \subseteq [0,1]$ *is monotonic non-increasing, continuous from the left, and* $r(x,u) = 0$ *for* $x > \omega_2$.

3. Type reduction of an IT2FS

Consider a crisp set $S_A(x)$, a classical fuzzy set $A(x)$, and an IT2FS $\tilde{A}(x)$, all of them related to the word A. The Type reduction process is simply the process of going from $\tilde{A}(x)$ to $S_A(x)$ using functions (or methods) namely f. This is:

$$\tilde{A}(x) \xrightarrow{f_1} A(x) \xrightarrow{f_2} S_A(x)$$

To do so, we introduce two methods: a centroid based method called the IASCO algorithm, and the YIR method as follows.

3.1. Centroid of an IT2FS

The IASCO algorithm proposed by Melgarejo in [12] is an improvement of the Enhanced Karnik-Mendel algorithm (See Karnik & Mendel in [22]) for Type-reduction of an Interval Type-2 fuzzy set. Given a set $\tilde{A}$, its centroid $C(\tilde{A})$ is composed by an interval set of centroids bounded by two values $\min\{C(\tilde{A})\} = C_l(\tilde{A})$ and $\max\{C(\tilde{A})\} = C_r(\tilde{A})$. Every point enclosed into $C(\tilde{A}) = [C_l(\tilde{A}), C_u(\tilde{A})]$ is also a possible centroid of $\tilde{A}$, so there is an infinite amount of centroids enclosed into $C(\tilde{A})$ (see Wu and Mendel [22], [23], Mendel and Liu [18], Karnik and Mendel [10], and Melgarejo [3]), as follows:

$$C(\tilde{A}) = 1/[c_l(\tilde{A}); c_r(\tilde{A})] \tag{7}$$

where $\tilde{A}$ is an interval Type-2 fuzzy set, $c_l(\tilde{A})$ and $c_r(\tilde{A})$ are the upper and lower centroids. The main equations of the Enhanced Karnik-Mendel (EKM) algorithm for computing $C(\tilde{A})$ (see Wu & Mendel [22]) are provided as follows:

$$c_l(\tilde{A}) = \frac{\sum_{i=1}^{N} x_i \underline{f}_i \; + \; \sum_{i=1}^{k} x_i \left(\overline{f}_i - \underline{f}_i\right)}{\sum_{i=1}^{N} \underline{f}_i \; + \; \sum_{i=1}^{k} \left(\overline{f}_i - \underline{f}_i\right)} \tag{8}$$

$$c_r(\tilde{A}) = \frac{\sum_{i=1}^{N} x_i \overline{f}_i \; - \; \sum_{i=1}^{k} x_i \left(\overline{f}_i - \underline{f}_i\right)}{\sum_{i=1}^{N} \overline{f}_i \; - \; \sum_{i=1}^{k} \left(\overline{f}_i - \underline{f}_i\right)} \tag{9}$$

where $\overline{f}$ and $\underline{f}$ are the UMF and LMF of $\tilde{A}$.

Other authors like Melgarejo [12] proposed the algorithm IASCO (Iterative Algorithm With Stop Condition) which improves the computation of $C(\tilde{A})$, as shown as follows:

$$D_0 = \sum_{i=1}^{N} x_i \underline{f}_i \tag{10}$$

$$P_0 = \sum_{i=1}^{N} \underline{f}_i \tag{11}$$

$$c_{min} = x_N \tag{12}$$

starting with $k = 0$, increasing $k = k + 1$ and computing D_0 and P_0 for each increment, then we have

$$D_k = D_{k-1} + x_k \left[\overline{f}_k - \underline{f}_k\right] \tag{13}$$

$$P_k = P_{k-1} + \left[\overline{f}_k - \underline{f}_k\right] \tag{14}$$

$$c_l(k) = \frac{D_k}{P_k} \tag{15}$$

if $c_l(k) \leq c_{min}$ then $c_{min} = c_l(k)$ otherwise stop and do $c_l = c_{min}$.

In a similar way, to find c_r from (9) the recursions are:

$$D_0 = \sum_{i=1}^{N} x_i \bar{f}_i \tag{16}$$

$$P_0 = \sum_{i=1}^{N} \bar{f}_i \tag{17}$$

$$c_{max} = x_N \tag{18}$$

starting with $k = 0$, increasing $k = k + 1$ and computing D_0 and P_0 for each increment, then we have

$$D_k = D_{k-1} - x_k[\bar{f}_k - \underline{f}_k] \tag{19}$$

$$P_k = P_{k-1} - [\bar{f}_k - \underline{f}_k] \tag{20}$$

$$c_r(k) = \frac{D_k}{P_k} \tag{21}$$

if $c_r(k) \geqslant c_{max}$ then $c_{max} = c_l(k)$ otherwise stop and do $c_r = c_{max}$

If the above condition is satisfied at some point stops the iteration cycle; the latest results are those for $c_l(\tilde{A})$ and $c_r(\tilde{A})$.

3.2. Yager Index rank for IT2FNs

Ronald Yager [24] has proposed one of the most important ranking methods for fuzzy sets. Based on the works of Figueroa-García & Pachón-Neira [6], Chaudhuri & Rosenfeld [4], and Hung & Yang [8] regarding α levels for computing distances, we present the Yager Index for IT2FNs as follows:

$$I(\tilde{A}) := \frac{1}{2}\int_{[0,1]} [\check{A}^U_\alpha + \hat{A}^L_\alpha, \check{A}^L_\alpha + \hat{A}^U_\alpha]\, d\alpha, \tag{22}$$

in the continuous case, and

$$I(\tilde{A}) := \frac{1}{2}\sum_{i=1}^{n} \left[(\check{A}^U_{\alpha_i} + \hat{A}^L_{\alpha_i}),\ (\check{A}^L_{\alpha_i} + \hat{A}^U_{\alpha_i})/2\right] \Delta_{\alpha_i}, \tag{23}$$

in the discrete case, where $\Delta_{\alpha_i} = \alpha_i - \alpha_{i-1}$ is the size of the partition (also known as the step size of the Riemmann's integral).

YIR leads to an interval set defined as $I(\tilde{A}) := [I_l(\tilde{A}) + I_r(\tilde{A})]$ which is easier to obtain than $C(\tilde{A})$ in the sense that any IT2FN is α-convex, so the computation of $I(\tilde{A})$ leads in some cases to closed forms. There is a limitation when using YIR over discrete variables: there is no a guarantee of having convex α-cuts, so we encourage readers to use any interpolation and/or approximation method in cases where ${}^\alpha\tilde{A}$ leads to open intervals.

4. Methodology of comparison

The relation between $C(\tilde{A})$ and $I(\tilde{A})$ is measured via subsethood between sets (see Kosko [11], Nguyen & Kreinovich [20], and Figueroa-García, Chalco-Cano & Román-Flores [5]). Consider two sets $C(\tilde{A}), I(\tilde{A})$ the set equality and subsethood between $C(\tilde{A}), I(\tilde{A})$ are defined as follows:

$$d_{=} = \frac{|C(\tilde{A}) \cap I(\tilde{A})|}{|C(\tilde{A}) \cup I(\tilde{A})|}, \tag{24}$$

$$d_{\subseteq} = \frac{|C(\tilde{A}) \cap I(\tilde{A})|}{|C(\tilde{A})|} \tag{25}$$

To do so, we have defined the following measures to compare $C(\tilde{A}_i)$ to $I(\tilde{A}_i)$:

- Center of centroid $C_c(\tilde{A}) = (c_l(\tilde{A}) + c_r(\tilde{A}))/2$
- Length of centroid $l_c(\tilde{A}) = c_r(\tilde{A}) - c_l(\tilde{A})$
- Center of YIR $I_c(\tilde{A}) = (I_l(\tilde{A}) + I_r(\tilde{A}))/2$
- Length of YIR $l_I(\tilde{A}) = I_r(\tilde{A}) - I_l(\tilde{A})$
- Footprint of Uncertainty $FOU(\tilde{A}) = \int_x \bar{\mu}_{\tilde{A}} dx - \int_x \underline{\mu}_{\tilde{A}} dx$
- Set equality $d_{=,i}$ and subsethood $d_{\subseteq,i}$

Now, we have performed 1000 simulations of three different different IT2FNs:

- Triangular: $T^U(\overline{\breve{a}}, \bar{a}, \overline{\hat{a}}), T^L(\underline{\breve{a}}, \bar{a}, \underline{\hat{a}})$,
- Gaussian: $G^U(\bar{a}, \hat{a}), G^L(\bar{a}, \breve{a})$,
- Non symmetric triangular (see Figure 3).

We have selected those shapes due to applicability in control and decision making problems and its easiness to implement in real world applications (see Wu & Mendel [19], Mendel [14], [16]).

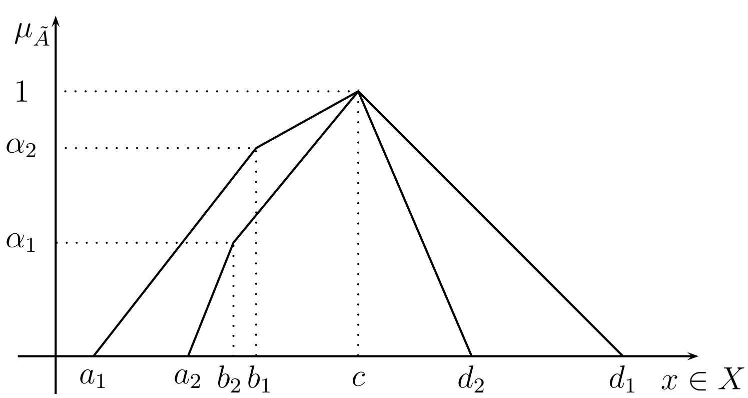

Figure 3. Non symmetric triangular IT2FN

A description of the procedure is shown in Procedure 1.

Procedure 1 Simulation methodology

for $i : 1 \to 1000$ **do**
 Simulate an IT2FN $\tilde{A}_i$
 Compute $C(\tilde{A}_i), I(\tilde{A}_i), C_c(\tilde{A}_i), I_c(\tilde{A}_i)$, and $FOU(\tilde{A}_i)$
 If $C_l < I_l$, set $y_{l,i} = 1$, 0 otherwise. If $C_r > I_r$, set $y_{r,i} = 1$, 0 otherwise
 Compute $d_{=,i}$ and $d_{\subseteq,i}$, set $D_{=,i} = d_{=,i} + d_{=,i-1}$ and $D_{\subseteq,i} = d_{\subseteq,i} + d_{\subseteq,i-1}$
 Compute FOU_i=$FOU(\tilde{A}_i)$+$FOU(\tilde{A}_{i-1})$
 Compute $l_{C,i}$ and $l_{I,i}$, if $l_{C,i} > l_{I,i}$ set $L_i = 1$, 0 otherwise
end for
Compute $\overline{FOU}, \bar{l}_C, \bar{l}_I, \bar{L}, \bar{D}_=, \bar{D}_\subseteq, \bar{C}_c, \bar{I}_c$

5. Results

After computing all measures shown in last section, we collect the results of the simulation procedure in Table I. This way, we present the average values for $d_=, d_\subseteq$ namely $\bar{D}_=, \bar{D}_\subseteq$ and the average values $\overline{FOU}, \bar{L}, \bar{C}_c, \bar{I}_c$ per type of IT2FN.

In Table I, *Center* is the average of all central parameters of all simulated IT2FNs. Note that $\bar{I}_c(\tilde{A})$ and $\bar{C}_c(\tilde{A})$ are very close to the central parameter of every set. This indicates that both methods are close to most possible granule of $\tilde{A}$ which is somehow an expected property. Another interesting finding is that centroid is as wider as the support of $\tilde{A}$ is, while YIR is as wider as the FOU is.

For triangular IT2FNs, a 71% of cases fulfill $I(\tilde{A}) \subseteq C(\tilde{A})$, a 83% of cases fulfill $C_l(\tilde{A}) \leqslant I_l(\tilde{A})$, a 88% of cases fulfill $C_r(\tilde{A}) \geqslant I_r(\tilde{A})$, and a 100% of cases $l_c(\tilde{A}) \geqslant l_I(\tilde{A})$.

For gaussian IT2FNs, a 100% of cases fulfill $I(\tilde{A}) \subseteq C(\tilde{A})$, a 100% of cases fulfill $C_l(\tilde{A}) \leqslant I_l(\tilde{A})$, a 100% of cases fulfill $C_r(\tilde{A}) \geqslant I_r(\tilde{A})$, and a 100% of cases $l_c(\tilde{A}) \geqslant l_I(\tilde{A})$. This happens due to the inherent symmetry of gaussian distribution which leads to centroid and YIR to converge to the same central value, and again YIR is less wide than centroid.

For non symmetric IT2FNs, a 57% of cases fulfill $I(\tilde{A}) \subseteq C(\tilde{A})$, a 98% of cases fulfill $C_l(\tilde{A}) \leqslant I_l(\tilde{A})$, a 59% of cases fulfill $C_r(\tilde{A}) \geqslant I_r(\tilde{A})$, and a 99% of cases $l_c(\tilde{A}) \geqslant l_I(\tilde{A})$. In this case YIR is still less wide than centroid, but due to the asymmetry of its left shape the YIR is greater than centroid.

Table I. Main results of the experiments over each set shape

Shape	Center	$\bar{C}_c(\tilde{A})$	$\bar{I}_c(\tilde{A})$	$\bar{l}_c(\tilde{A})$	$\bar{l}_I(\tilde{A})$	$\overline{FOU}(\tilde{A})$	$\bar{L}$	$\bar{D}_=$	$\bar{D}_\subseteq$
Triangular	29.87	30.10	30.04	6.99	5.11	10.21	0.56	0.685	0.699
Gaussian	75.17	75.17	75.17	5.74	3.06	6.12	0.07	0.602	0.602
Non-sym.	43.66	40.05	41.33	10.6	6.69	16.96	0.23	0.99	0.98

In Table I, $\bar{L}$ is the average of times that $C_c(\tilde{A}) > I_c(\tilde{A})$ which means that even when the centers of YIR and centroid are similar, there are small numerical differences. For instance, only a 7% of all gaussian simulations show this behavior (due to small numerical differences) even when all simulated gaussian IT2FNs were symmetric. It is also clear that this behavior changes in non symmetric IT2FNs.

In general, both centroid and YIR represent the central value of $\tilde{A}$ in a good way with some numerical differences. While YIR is a function of α, the centroid is a function of the support of $\tilde{A}$ which finally leads to different results.

6. Concluding Remarks

From a computational point of view, the IASCO and EKM algorithms use expensive routines whose results are wider measures than the YIR. In most of cases, the centroid is a wider measure than the YIR. For gaussian IT2FNs, a 100% of times the centroid contains the YIR, while for triangular IT2FNs only a 71% the centroid contains YIR. It seems that the FOU of $\tilde{A}$ has a relationship to $I(\tilde{A}), C(\tilde{A})$ since as large the FOU is, as large $I(\tilde{A}), C(\tilde{A})$ are. Future works will corroborate those results.

In general, the YIR seems to be an easier way to compute the expected value of $\tilde{A}$ than $C(\tilde{A})$ due to its simplicity and good relationship to its FOU. Evidently, the shape of the set infers on the properties of $I(\tilde{A}), C(\tilde{A})$, so we recommend to keep in mind that $C(\tilde{A})$ is wider than $I(\tilde{A})$. For gaussian shapes, both $I(\tilde{A}), C(\tilde{A})$ obtains the same $I_c(\tilde{A}), C_c(\tilde{A})$, for non symmetric triangular shapes most of times $I_c(\tilde{A}) > C_c(\tilde{A})$, and for triangular shapes near a half of cases $I(\tilde{A}) > C(\tilde{A})$.

For practical applications, we recommend the use of YIR over centroid since YIR has closed equations while IASCO and EKM algorithms are iterative methods. Note that YIR has been designed for IT2FNs while IASCO and EKM algorithms have been designed for all kinds of IT2FSs.

This way, YIR has a great potential in fuzzy optimization, resolution of fuzzy equations, hardware implementation of fuzzy controllers, and other applications where iterative computations are neither allowed nor feasible.

Future work

Other experiments can be performed in the future to see other properties of $I(\tilde{A}), C(\tilde{A})$ in other fields such computation of fuzzy functions, fuzzy optimization, fuzzy decision making, etc. Also, some theoretical differences between YIR and centroid could be interesting to be analyzed such as the relationship between $I(\tilde{A})$ and $FOU(\tilde{A})$ and the sizes of $I(\tilde{A}), C(\tilde{A})$.

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