Tekhnê
January - June 2018, Vol. 15, No. 1, pp. 21 – 26

ISSN 1692-8407

Modeling and design of DC/DC converters

Modelo y diseño de convertidores CC/CC

Lucas Hincapié C.
Universidad Distrital Francisco José de Caldas
`hegehytor6@hotmail.com`

Edwin I. Garavito P.
Universidad Distrital Francisco José de Caldas
`edwgapa@hotmail.com`

This paper specifies the type of systems that will be studied, the DC/DC power converters, as well as the type of control that will be applied to them to improve their dynamic response. For this purpose, the mathematical models to be used for the analysis and design of control loops of the Buck/Boost model will be introduced. The history of non-linear control applied to DC/DC converters will also be reviewed and circuit On and Off time analysis and its variation in these states will be established.

Keywords: Average model, closed-loop control, power converter, rectifier, regulator

En este artículo se precisa el tipo de sistemas que serán estudiados, los convertidores de potencia CC/CC, así como el tipo de control que se les aplicará para mejorar su respuesta dinámica. Para ello, se introducirán los modelos matemáticos que se emplearán para el análisis y diseño de lazos de control del modelo Buck/Boost. También se hará un recorrido por los antecedentes del control no lineal aplicado a los convertidores CC/CC y se establecerán análisis de tiempo On y Off de circuito y su variación en dichos estados.

Palabras clave: Control en lazo cerrado, convertidor de potencia, modelo promedio, rectificador, regulador

Article typology: Research

Date manuscript received: December 6, 2017
Date manuscript acceptance: December 15, 2017

Research funded by: Universidad Distrital Francisco José de Caldas.

How to cite: Hincapié, L., Garavito, E. (2018). *Modeling and design of DC/DC converters*. Tekhnê, 15(1), 21 -26.

Introduction

The DC/DC power converters are DC voltage regulator circuits characterized by high-frequency switching operation (at least very large compared to the frequency of the public power grid) which gives them greater efficiency compared to linear regulators but turns their circuit into a non-linear one making their closed-loop control a real design challenge (Bazurto & Martínez, 2015; Li & Parsa, 2018; Ochoa, Rodriguez, & Martínez, 2017).

In this document, an ideal analysis of a DC/DC circuit, the Buck/Boost converter, is made to establish the basic criteria for the design and implementation of analog non-linear controls that improve the dynamic performance of the power converter, particularly when they include the reconstruction of the current waveform (active correction of the Power Factor) (Braslavsky, Plotnikov, & Valtchev, 2016; Callegaro, Ciobotaru, Pagano, Turano, & Fletcher, 2018; Mashinchi Mahery & Babaei, 2013).

The final behavior of the fed back systems is validated both at the simulation level and by performing various physical implementations of the controller and corrector or PFC (Martínez & Gómez, 2004). Preliminary results show that with this basic analysis and design an experimental prototype can be obtained where the presence of noise or disturbances does not significantly affect the waveforms and output regulation levels (Miao, Wang, & Ma, 2016; Takagi & Fujita, 2018; Zhifu, Yupu, & Yinan, 2017).

The document is structured as follows. The following section reviews the general concepts of DC/DC converters, the mathematical models, and the types of control that will serve as a theoretical basis. The results are then presented in the laboratory for a low power prototype (Tan & Hoo, 2015).

Model of the buck/boost converter

The basic topologies of DC/DC converters are buck, boost, and buck/boost. These three controllers are characterized by having a single transistor (controlled switch) and have a single-stage conversion. The output power is usually small, in the order of tens of watts.

The buck/boost converter is a converter commonly used for the inversion of the output voltage polarity concerning the input. Its output voltage to the input can be either reducing or increasing, depending on the desired duty cycle. The topology of this converter is shown in Fig. 1.

The DC/DC converter shown in the Fig. 1 is characterized by the fact that it has a direct electrical connection between input and output. It also has an inductor and a capacitor. The latter serve as energy storage during each switching cycle of the transistor.

The transistor is used as a controlled switch and the diode as an uncontrolled switch to carry out the switching. In an ideal switch, the voltage in the ON position is zero, as is the current during the OFF interval. Therefore, ideally, it never dissipates energy because the product $v \times i$ is always zero. Active switches such as the transistor turn on or off in response to an applied signal, and passive switches (diodes) have a non-linear *v-i* characteristic function that operates according to the polarization of the device. Voltage drops in the switches can be affected when the input and output voltages of the converter are low.

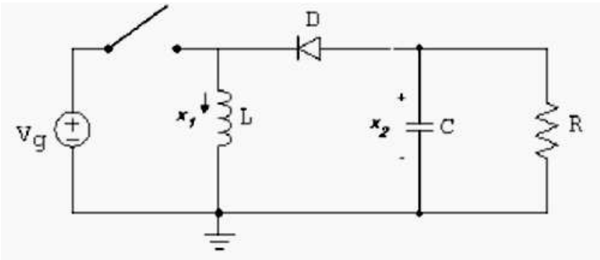

Figure 1. Buck/Boost converter.

Simplified switching model

The converter is said to have two operating states depending on the control signal on the transistor and its corresponding behavior (Fig. 2). When the activation signal is high, the transistor closes, which together with the choke forces the diode to remain open (Fig. 3). During this interval, the choke absorbs energy, and the current increases linearly. The load current is supplied by the output capacitor, which is why its design considers both the highest T_{ON} value, the size of the load, and the maximum voltage ripple value at the output. During T_{OFF} the control signal opens the transistor and the operation is reversed (Fig. 4). The voltage on the diode is reversed causing it to conduct and transfer energy to the output capacitor and the load. The critical inductance value is related to the amount of energy required during this interval.

Let's analyze the circuit during T_{ON}.

$$V_l = V_{in} - V_{R_{ON}} \tag{1}$$

$$L\frac{di}{dt} = V_{in} - i_L R_{ON} \tag{2}$$

$$\frac{di}{dt} = \frac{V_{in}}{L} - \frac{i_L R_{ON}}{L} \tag{3}$$

$$i_C = \frac{V_o}{R_L} \tag{4}$$

$$C\frac{dv}{dt} = -\frac{V_o}{R_L} \tag{5}$$

$$\frac{dv}{dt} = -\frac{V_o}{CR_L} \tag{6}$$

ISSN 1692-8407

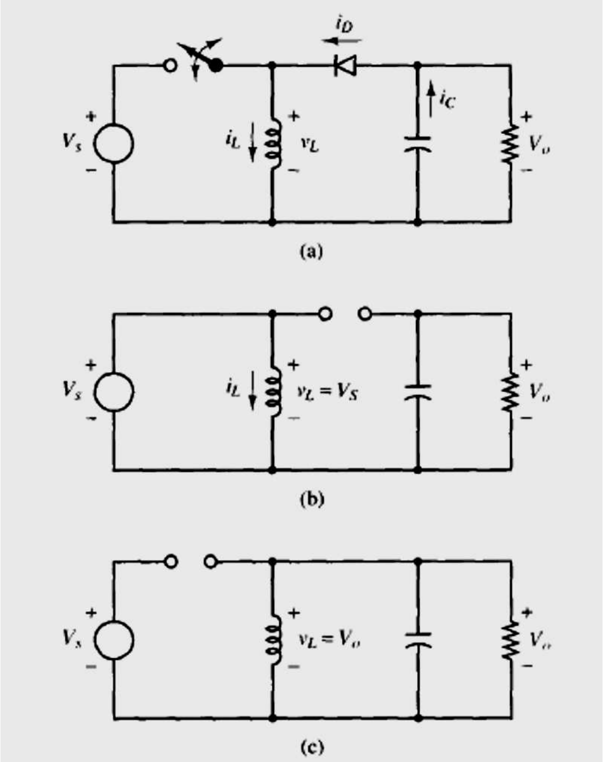

Figure 2. Buck/boost converter models.

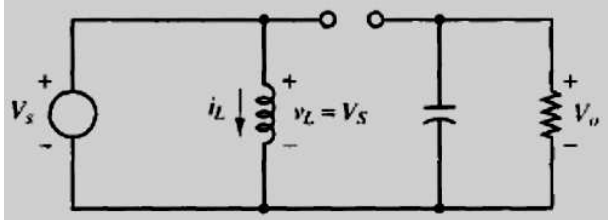

Figure 3. T_{ON} circuit analysis.

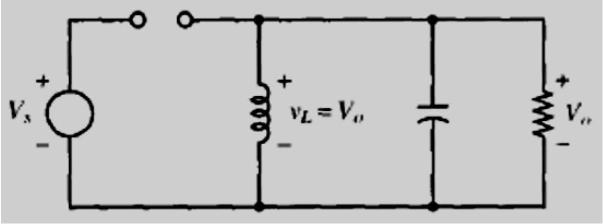

Figure 4. T_{OFF} circuit analysis.

During T_{OFF}.

$$V_l = V_C = V_o \tag{7}$$

$$L\frac{di}{dt} = V_o \tag{8}$$

$$\frac{di}{dt} = \frac{V_o}{L} \tag{9}$$

$$i_C = i_L - i_{R_L} \tag{10}$$

$$C\frac{dv}{dt} = i_L - \frac{V_o}{R_L} \tag{11}$$

$$\frac{dv}{dt} = \frac{i_L}{C} - \frac{V_o}{CR_L} \tag{12}$$

From equations (3) and (9):

$$D\left(\frac{V_{in}}{L} - \frac{i_L R_{ON}}{L}\right) + (1 - D)\left(\frac{V_o}{L}\right) = \frac{di}{dt} \tag{13}$$

From equations (6) and (12):

$$D\left(-\frac{V_o}{CR_L}\right) + (1 - D)\left(\frac{i_L}{C} - \frac{V_o}{CR_L}\right) = \frac{dv}{dt} \tag{14}$$

Dynamic model

From the definition of the duty cycle we have:

$$T = D + (1 - D) \tag{15}$$

For the system of equations:

$$\begin{gathered}\dot{X}(t) \approx AX(t) + B\\ \dot{X}(t) \approx A_1X(t) + B_1\\ \dot{X}(t) \approx A_2X(t) + B_2\end{gathered} \tag{16}$$

$$A_1 = \begin{bmatrix} -\frac{R_{ON}}{L} & 0 \\ 0 & \frac{1}{R_2C} \end{bmatrix} \tag{17}$$

$$A_2 = \begin{bmatrix} 0 & \frac{1}{L} \\ -\frac{1}{C} & \frac{1}{R_LC} \end{bmatrix} \tag{18}$$

$$B_1 = \begin{bmatrix} \frac{V_{in}}{L} \\ 0 \end{bmatrix} \tag{19}$$

$$B_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{20}$$

With the following expression a bilinear model can be obtained, having found that the behavior is linear in both states. This is an approximation of the model applied to the Buck/Boost converter in order to later determine its state-space function.

$$\dot{X} = [A_1D + A_2(1 - D)]X + [B_1D + B_2(1 - D)] \tag{21}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} DR_{ON} & \frac{(1-D)}{L} \\ \frac{(D-1)}{C} & -\frac{1}{RC} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \frac{2V_{in}}{3L} - \frac{(3D-1)V_{in}}{3L\left(3D^2-3D-1\right)} \\ \frac{DV_{in}(2D-1)}{RC\left(3D^2-3D-1\right)} \end{bmatrix} \tag{22}$$

Static model

The static model is based on equations that describe both conduction and non-conduction behavior. The equations are determined by the main energy storage element and in the dynamics of the converter. The analysis is performed on the inductance, which is a constant and can be expressed mathematically in both transistor operating intervals.

During T_{ON}:

$$\frac{\Delta i_L}{\Delta t} = \frac{\Delta i_L}{DT} = \frac{V_{in}}{L} - i_L \frac{R_{ON}}{L} \tag{23}$$

$$\Delta i_L = \left(\frac{V_{in}}{L} - i_L \frac{R_{ON}}{L}\right) \times DT \tag{24}$$

During T_{OFF}:

$$\frac{\Delta i_L}{\Delta t} = \frac{\Delta i_L}{(1-D)\,T} = \frac{V_{in}}{L} \tag{25}$$

$$\Delta i_L = \left(\frac{V_o}{L}\right) \times (1-D)\,T \tag{26}$$

Performing current balance on the choke.

$$\Delta i_L \times DT - \Delta i_L \times (1-D) \times T = 0 \tag{27}$$

$$DT\left(\frac{V_{in}}{L} - \frac{i_L R_{ON}}{L}\right) + (1-D)\,T\left(\frac{V_o}{L}\right) = 0 \tag{28}$$

$$V_o = \left(\frac{D}{1-D}\right)(i_L \times R_{ON} - V_{in}) \tag{29}$$

Checking through Faraday's law.

$$\int_0^T v_L(t)\,dt = (V_{in} - i_L R_{ON})\,DT + V_o\,(1-D)\,T \tag{30}$$

$$V_o = \left(\frac{D}{1-D}\right)(i_L \times R_{ON} - V_{in}) \tag{31}$$

Using the expression found, the behavior of the input and output voltage can be simulated depending on the variation of the signal's useful cycle. This response was simulated and is shown in Fig. 5.

The mathematical DC/DC models shown above can also be classified according to their mode of operation, depending on the continuity or otherwise of the current flowing through the choke. In this way, when the current is always greater than zero during the entire switching period, the converter will work in continuous mode, and discontinuous mode if the current in the choke is canceled for any instant.

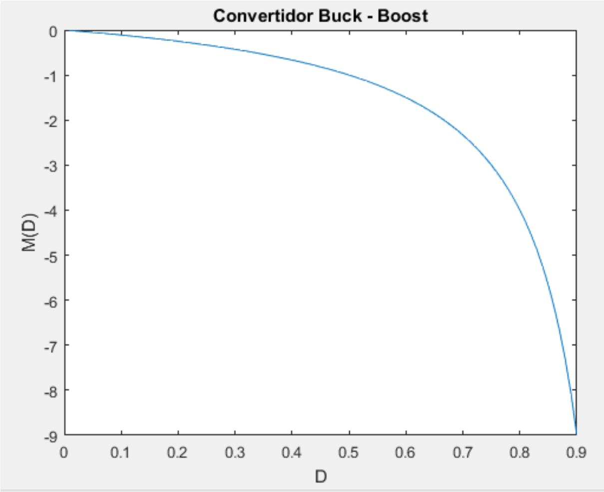

Figure 5. Behaviour curve according to the variation of the duty cycle.

Prototype behaviour

The dynamic behavior of the drive is described in two operating states, which consist of the position of the Q and D switches that are periodically turned on and off from a binary signal. The regulator is capable of maintaining a constant voltage above or below the input voltage according to the duty cycle value D (Figs. 6, 7 and 8).

The figure also reflects the behavior of the input voltage and maximum current variation. The input voltage was assumed for simulation purposes to be ±30%. The load regulation is defined as the difference between V_o measured when the load is minimum ($R = R_{max}$) and V_o when the load is maximum ($R = R_{min}$), divided by the nominal V_o.

$$R_C\% = \left(\frac{V_o R_{max} - V_o R_{min}}{V_{o(nominal)}} \times 100\right) \tag{32}$$

Conclusions

In this article, we analyze the construction and the Buck/Boost model reconstructing integrally the equations of the model that presented the particularity of establishing different design criteria from its conceptual way to its practical way using the simulations. Also, the article studies the stability in the large signal of this switched regulator for the different states analyzed, guaranteeing a steady-state error of zero and demonstrating that it can be easily implemented with analog integrators and multipliers.

The procedure established for the development of this article allowed the basic static and dynamic design for a Buck/Boost converter operating in continuous conduction mode. The response of the simulation shows the dynamic behavior of the state variables of the proposed circuit.

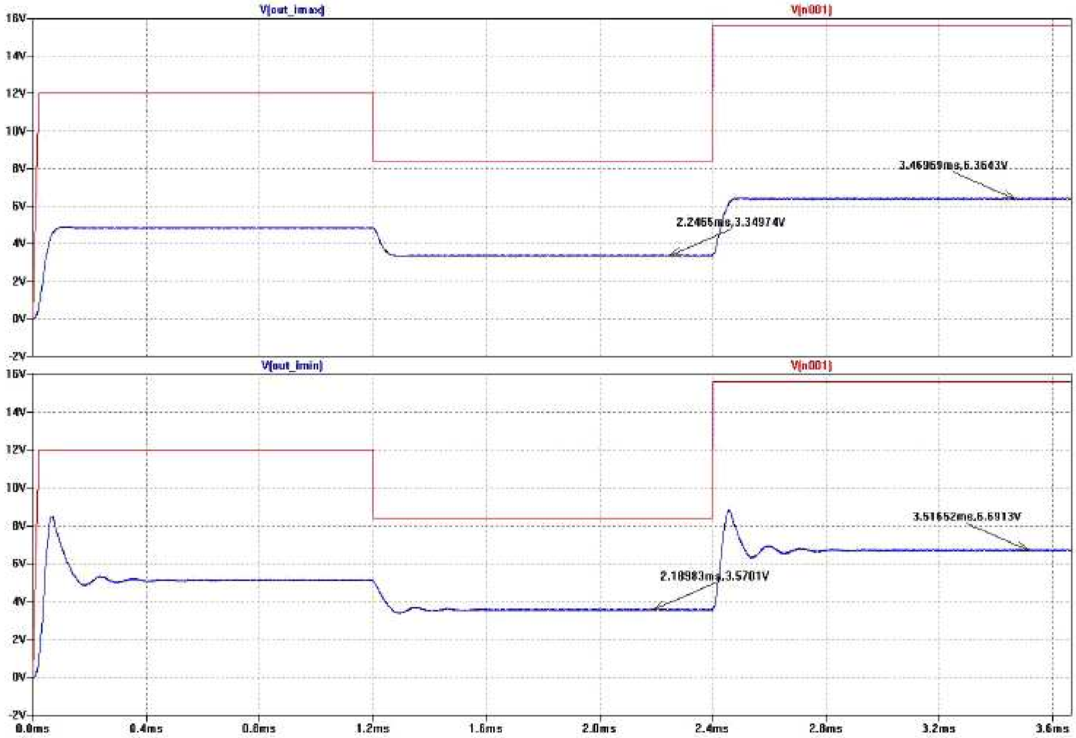

Figure 6. Maximum current and input voltage variation.

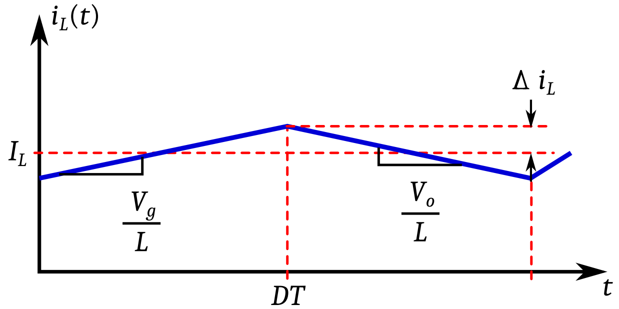

Figure 7. Inductor current waveform.

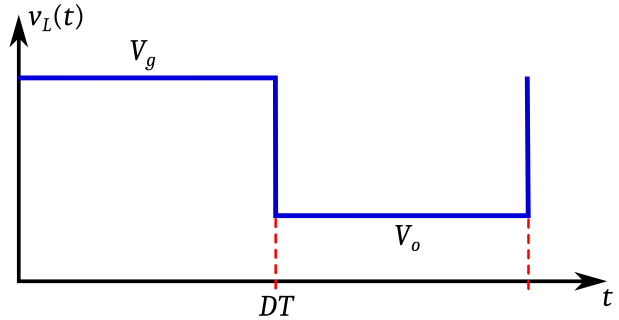

Figure 8. Inductor voltage waveform.

References

Bazurto, J., & Martínez, F. (2015). Estudio y diseño de fuente electrica para aplicaciones de electroerosion. *Tecnura*, *19*(Extra 2), 99-106. Retrieved from `https://dialnet.unirioja.es/servlet/articulo?codigo=6371453` (Publisher: Universidad Distrital Francisco José de Caldas Section: Tecnura: Tecnología y Cultura Afirmando el Conocimiento)

Braslavsky, I., Plotnikov, I., & Valtchev, S. (2016). The mathematical modeling of alternating current electric drive with DC-DC converter and ultracapacitors. In *2016 IEEE International Power Electronics and Motion Control Conference (PEMC)* (p. 511-515).

Callegaro, L., Ciobotaru, M., Pagano, D. J., Turano, E., & Fletcher, J. E. (2018). A Simple Smooth Transition Technique for the Noninverting Buck–Boost Converter. *IEEE Transactions on Power Electronics*, *33*(6), 4906-4915. (Conference Name: IEEE Transactions on Power Electronics)

Li, T., & Parsa, L. (2018). Design, Control, and Analysis of a Fault-Tolerant Soft-Switching DC–DC Converter for High-Power High-Voltage Applications. *IEEE Transactions on Power Electronics*, *33*(2), 1094-1104. (Conference Name: IEEE Transactions on Power Electronics)

Martínez, F., & Gómez, D. (2004). Corrección activa del factor de potencia en cargas no lineales. *Tecnura*, *7*(14), 40-47. Retrieved from `https://revistas.udistrital.edu.co/index.php/Tecnura/`

article/view/6184

Mashinchi Mahery, H., & Babaei, E. (2013). Mathematical modeling of buck–boost dc–dc converter and investigation of converter elements on transient and steady state responses. *International Journal of Electrical Power & Energy Systems*, *44*(1), 949-963. Retrieved from `http://www.sciencedirect.com/science/article/pii/S0142061512004838`

Miao, S., Wang, F., & Ma, X. (2016). A New Transformerless Buck–Boost Converter With Positive Output Voltage. *IEEE Transactions on Industrial Electronics*, *63*(5), 2965-2975. (Conference Name: IEEE Transactions on Industrial Electronics)

Ochoa, Y., Rodriguez, J., & Martínez, F. (2017). Low cost regulation and load control system for low power wind turbine. *Contemporary Engineering Sciences*, *10*(28), 1391-1399.

Takagi, K., & Fujita, H. (2018). Dynamic Control and Performance of a Dual-Active-Bridge DC–DC Converter. *IEEE Transactions on Power Electronics*, *33*(9), 7858-7866. (Conference Name: IEEE Transactions on Power Electronics)

Tan, R. H., & Hoo, L. Y. H. (2015). DC-DC converter modeling and simulation using state space approach. In *2015 IEEE Conference on Energy Conversion (CENCON)* (p. 42-47).

Zhifu, W., Yupu, W., & Yinan, R. (2017). Design of closed-loop control system for a bidirectional full bridge DC/DC converter. *Applied Energy*, *194*, 617-625. Retrieved from `http://www.sciencedirect.com/science/article/pii/S0306261916317445`