Proposal of a fault estimation using coprime fractions
Propuesta de un estimador de fallas usando fracciones coprimas

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Abstract: detection, diagnostic and fault isolation systems FDD generate signals (residuals) containing details of the fault. The state estimation is a common procedure for residual generation when it used analytically. In this article, the proposed hypothesis is to use the theory of coprime fractions to generate an estimator of the output; it competes with some classic Kalman, Luenberger, Lyapunov and Jordan estimators and thus analyses their execution times and accuracy rates by the mean square error in presence of additive failures. The results are validated on a FDD system implemented on a homogenization tank in a closed loop using the standard process control OPC and physically simulating a sensor fault in a RTD.

KeyWords: Canonical forms, coprime fraction, fault, FDD, multivariable estimator, residuals.

Resumen: Los sistemas de detección, diagnóstico y aislamiento de fallas (FDDI), generan señales (residuos) que contienen detalles de la falla. La estimación de estado es un procedimiento común para la generación de residuos cuando se usa redundancia analítica. En este artículo, la hipótesis propuesta es usar la teoría de fracciones coprimas para generar un estimador de salida; este compite con algunos estimadores clásicos como Kalman, Luemberger, Lyapunov y Jordan, con ellos se analizan los tiempos de ejecución y tasas de precisión en la estimación

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usando el error medio cuadrático en presencia de fallas aditivas. Los resultados son validados sobre un sistema FDDI implementado sobre un tanque de homogenización en lazo cerrado usando el estándar para control de procesos OPC y simulando físicamente una falla en el sensor RTD de salida.

Palabras clave: Formas canónicas, fracciones coprimas, falla, FDD, estimador multivariable, residuos.

1. Introduction

The reliability required in many systems or processes have created the necessity of detecting abnormal conditions while systems or processes are operating. A fault in a system or processes is considered as a not-allowable deviation that can be detected by an appropriated signal evaluation [1-4]. Fault detection, diagnosis, and isolation schemes must be applied in order to reliably support human operators in the management of malfunctions [5]. Fault diagnosis consists of fault detection and fault isolation. Fault detection determines a timely detection of an abnormal event and fault isolation determining where the fault is diagnosing its causal origins [6].

Many methods have been applied to fault detection, mainly with estimators or observers of faults. In this paper, we present a proposal of an estimator using coprime fractions in comparison between some estimators for fault detection. The response of coprime fraction under comparison tests is fast and offers low error estimation.

The rest of paper is organized as follows: Section 2 presents some basics on fault detection and diagnostics. Section 3 provides an overview of the some techniques of fault detection estimators. Section 4 presents the comparisons of the techniques of estimators. Section 5 contains the proposal; Section 6 shows an application example; and finally, Section 7 presents the concluding remarks of the study.

2. Fault detection and diagnostics system

For fault detection, according to Hermann [1], the main task of the different mathematical models is the detection of faults in the processes, actuators, and sensors using dependencies between the various measurable signals in the system. These dependencies generally are expressed by means of mathematical models, linking input signals to output signals and residuals signals that are the result of detection systems, designed parameters, or estimated state variables. Some signals (1) mentioned above show dependencies between input signals $u(t)$, output signals $y(t)$, state variables $x(t)$, with $w(t)$ and $v(t)$ disturbances process signals and $f_l\ (t)$ and $f_m(t)$ additive process faults signals. Each of these signals is called 'feature of the system' and is compared with the nominal value expected by changing detection methods, the result is analytical symptoms, which are the basis for fault diagnosis (see Figure 1).

$$\dot{x}(t) = Ax(t) + Bu(t) + Ww(t) + Lf_l(t)$$

$$y(t) = Cx(t) + Bu(t) + Vv(t) + Mf_m(t) \quad (1)$$

Faults are usually classified into three types [2]: additive measurement failures, additive process faults, and multiplicative process faults. The first described well deviations in measurements provided by the sensors, they are also used to describe a malfunction in the actuators, the second are disturbances in the plant, and the third describe the abrupt changes of the parameters of the plant that appear as deterioration of equipment interconnected to the plant.

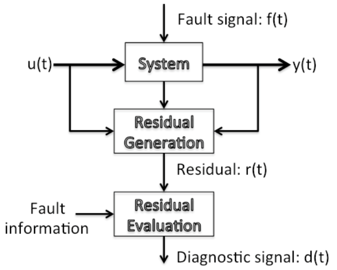

Figure 1. Structure of a fault detection and diagnostics system. Source: [3].

Procedures based on analytical redundancy for residual generation can be divided generally in the procedures based on mathematical models and those based on artificial intelligence. One type of residual generation based on mathematical models is through output observers: the procedure is by a linear transformation in (1) which generates new state variables $\boldsymbol{\xi}(\boldsymbol{t}) = \boldsymbol{T_1}\,\boldsymbol{x}(\boldsymbol{t})$ (2, 3). The residual $\boldsymbol{r}(\boldsymbol{t})$ in (4) can be designated in such a way that it is independent of the unknown input $\boldsymbol{v}(\boldsymbol{t})$ and state $\boldsymbol{x}(\boldsymbol{t})$ and $\boldsymbol{u}(\boldsymbol{t})$ by special matrices $\boldsymbol{C\xi}$ and $\boldsymbol{T_2}$. Residual depends only on additive faults $\boldsymbol{f_l}\,(\boldsymbol{t})$ and $\boldsymbol{f_m}\,(\boldsymbol{t})$, graphically in figure 2 it is shown by the output observer.

$$\dot{\hat{\xi}}(t) = A_\xi \hat{\xi}(t) + B_\xi u(t) + G_\xi y(t) \quad (2)$$

$$\eta(t) = C_\xi \hat{\xi}(t);\ \hat{\eta}(t) = T_2 y(t) \quad (3)$$

$$r(t) = \eta(t) - \hat{\eta}(t) \quad (4)$$

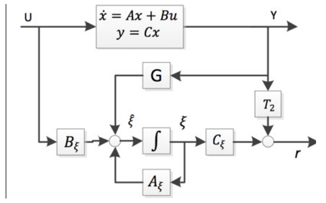

Figure 2. Residual generation with output observer. Source: [1].

2.1. Coprime Fractions

Space state multivariable models that have more than one input p or more than one output q, can be transforms in a matrix transfer function (MTF) [7]. Every $q \times p$ proper rational matrix $\dot{\hat{G}}(s)$ can be expressed as a matrix polynomial fraction (MPF) like (5) called left MPF or as (6) called right MPF. Some authors like Chen [8], Kauvaritakis [9], Graselli [10] and others, they have analyzed algorithms for transformation of the models, and they have established characteristics of multivariable systems from MPF models. If $D(s) \in \mathbb{R}^{q \times r}$, $N(s) \in \mathbb{R}^{p \times q}$ are coprime fractions, we can find minimal realizations with different methods [11-13].

$$\tilde{G}(s) = D(s)^{-1}N(s) \quad (5)$$

$$\tilde{G}(s) = N(s)D(s)^{-1} \quad (6)$$

One of the developments in order to find a realization in observable canonical form (10) starts from the lower triangular matrix D_{hc} with the highest coefficients (hc) per row (7), hence the terms of the matrix $\tilde{C}$ of the realization (8) will appear.

$$\lim_{s \to \infty} \hat{G}(s) = D_{hc} N_{hc} \quad (7)$$

$$D_{hc}^{-1} = \begin{bmatrix} 1 & o \\ c_{21} & 1 \end{bmatrix} \quad (8)$$

$$L(s) = \begin{bmatrix} s^{v2-1} & \cdots 1 & 0 \cdots & & 0 \\ \vdots & & s^{v2-1} & \cdots 1 & \vdots \\ 0 & & \cdots 0 & s^{v-1} & \cdots 1 \end{bmatrix}' \quad (9)$$

The coefficients of the result of $D_{lc}D_{hc}^{-1}$ are organized in $\tilde{A}$ matrix. Finally, the $\tilde{B}$ matrix formed with coefficients $N(s)L(s)$, v is called a pseudo state. There is a dual case of right MPF, where the realization ends in a controllable way [11].

$$\tilde{A} = \begin{bmatrix} \left[\begin{array}{c|c} \frac{0}{I} & \begin{matrix} x \\ x \end{matrix} \end{array}\right]^{v_1 x v_1} \times & [\dots] & \left[\begin{array}{c|c} 0 & \begin{matrix} x \\ x \end{matrix} \end{array}\right] \\ \left[\begin{array}{c|c} 0 & \begin{matrix} x \\ x \end{matrix} \end{array}\right] & [\ddots] & \left[\begin{array}{c|c} 0 & \begin{matrix} x \\ x \end{matrix} \end{array}\right] \\ \left[\begin{array}{c|c} 0 & \begin{matrix} x \\ x \end{matrix} \end{array}\right] & [\dots] & \left[\begin{array}{c|c} \frac{0}{I} & \begin{matrix} x \\ x \end{matrix} \end{array}\right]^{v_r x v_r} \end{bmatrix}$$

$$\tilde{C} = \begin{bmatrix} \left[\begin{array}{c|c} 0 & \begin{matrix} 1 \\ x \end{matrix} \end{array}\right] & [\dots] & \left[\begin{array}{c|c} 0 & \begin{matrix} 0 \\ 1 \end{matrix} \end{array}\right] \end{bmatrix} \quad (10)$$

3 Output Estimators

3.1 Kalman Estimator

This filter is the principal algorithm to estimate dynamic systems specified in state-space form using the covariance of error R and perturbation Q. The Kalman estimator is a set of mathematical equations (11, 12) that provides an efficient computational (recursive) solution of the least-squares method [14-17]. The goal is to find an unbiased minimum variance lineal estimator of the state at time t with base in available information at time before $(t-1)$.

$$P_k^- = AP_{k-1}A^T + Q \quad (11)$$

$$G_k = P_k^- C^T [CP_k^- C^T + R]^{-1} \quad (12)$$

3.2 Luenberger Estimator

The idea on which the observer is based is based on generating a "clone" system of the original, which is itself the internal state that can be measured directly. If the original system and its clone are subjected to the same stimuli (the input), you can expect that as time passes, they begin to behave the same way because their internal states tend to look more and more alike (this works as long as the original system and its clone are stable). Thus, the internal state of the clone can be used as an approximation of the internal state of the original system.

3.3 Lyapunov Estimator

This is a simple algorithm if you can find a diagonal square matrix $(n-q) \times (n-q)F$ easily that has no eigenvalues in common with those of A. Then you can solve the Lyapunov equation (13) choosing arbitrary $(n-q) \times qG_{ly}$ such that (F, G_{ly}) is controllable.

$$TA - FT = G_{ly}C \quad (13)$$

3.4 LQ Estimator

The optimal gain minimizes the quadratic cost function (14) with the covariance of noise and perturbation R, N, Q. The estimator should solve the Ricatti equation that determines the matrix of covariance S, the solution will be optimal if the pair (A, C) is stabilized and (A, T) detectable.

$$J(u) = \int_0^{\infty} (x^T Q x + u^T R u + 2x^T N u)\, dt \quad (14)$$

3.5 Jordan Design

In this Jordan or cyclic design, we change the multi-input problem into a single-input problem finding a vector v then applying the location of poles using a classical method $k_1 = vk$. A matrix A is called cyclic if and only if the Jordan form of A has one and only one Jordan block associated with each distinct eigenvalue [11]. If the system is not cyclic should find a feedback matrix k_2, which makes the cyclic system, the result $G_{cy} = k_1 + k_2$.

3.6 Canonical Form Design

The main idea is based on the transformation of a system in an observable canonical form

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e.g., No.4 (eq. 10), by means of a transformation matrix P. If the system is at minimum, you can transform the system in a controllable canonical form also, for example, assuming that controllability indexes are $v = [3\ 1]$ (15, 16) and the coefficients of the desired poles $\tilde{\alpha}$ with (17) finally is the feedback matrix for the observer.

$$\dot{x}(t) = \left[\begin{array}{ccc|c} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \alpha_{111} & \alpha_{112} & \alpha_{113} & \alpha_{121} \\ \hline \alpha_{211} & \alpha_{212} & \alpha_{213} & \alpha_{221} \end{array}\right] x(t) \quad (15)$$

$$y(t) = \left[\begin{array}{ccc|c} 0 & 0 & 1 & 0 \\ 0 & 0 & c_{12} & 1 \end{array}\right] x(t) \quad (16)$$

$$K_f = \left[\begin{array}{ccc|c} \alpha_{111} & \alpha_{112} & \alpha_{113} & \alpha_{121} \\ \alpha_{211} & \alpha_{212} & \alpha_{213} & \alpha_{221} \end{array}\right] + \left[\begin{array}{ccc|c} \tilde{\alpha}_{111} & \tilde{\alpha}_{112} & \tilde{\alpha}_{113} & 0 \\ 0 & 0 & 0 & \tilde{\alpha}_{221} \end{array}\right]$$

$$K_c = \begin{bmatrix} 1 & 0 \\ c_{12} & 1 \end{bmatrix}; G_{fc} = K_c K_f P \quad (17)$$

3.7 Methodology of Coprime Fraction Design

Classically, search matrix state feedback K is obtained from a system described in a controllable state space. After that and depending on the control law is applied to the system (1). The method based on canonical forms follows the classical method, needing to convert the system described in state space to fcc12 or fcc11 by a similarity matrix. This process is described in [11], by decomposition in r subsystems with horizontal coupling coefficients, in this case it is required; so the system is in form 12, this is because algorithmically it is difficult to know why or how the system is, when the system is in form 12 or 11 the algorithm returns to a matrix identity. In order to design an output observer, we need to start with an observable canonical form like fcoNo.4 (10). However, it does not matter if you start with a controllable form because it is coprime, see Figure 3 –notice that the model is a right MPF. G_{cf} The observer matrix is similar to that found in state feedback, because of the duality theorem. The matrix found by this method is always equal to the matrix found by canonical forms only if the coprime system is in observable canonical form, hence the results of the estimates are similar.

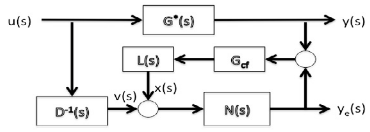

Figure 3. Observer scheme by coprime fractions. Source: [1].

In a first continuous test on a pilot plant see figure 4, the output estimators show us the velocity of estimation; the cyclic design is the fastest followed by Kalman, lq estimator, and coprime fraction. The estimator Luenberger and Lyapunov try to average the values, and the movements are slow. The response of coprime fractions for this test is in the middle, not fast, but not slow, neither with a big error in estimation.

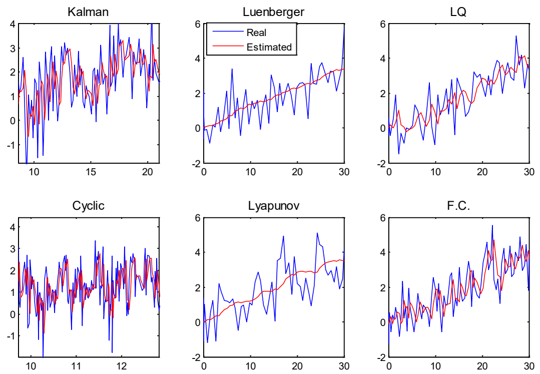

Figure 4. Output estimation. Source: own.

4. Diagnostics and Isolation of the Faults

The failures usually show a characteristic behavior for various components and usually originate from internal or external to system forms; therefore failure can be generated in sensors or actuators, hardware or structures that make up the system or system disturbances due to external sources [1]. The various faults can be distinguished by their shape (see figure 5), which can be systematic or random (fault 4 in Figure 5);

by its behavior over time it can be described as permanent, temporary, intermittent, or fluctuating.

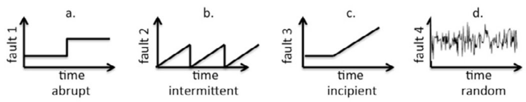

Figure 5. Different kind of Faults a) unit step, b) sawtooth, c) ramp and d) random signal. Source: [13].

Residual generations that obtain signals containing information only about the failures are called residuals. In the ideal case, the residuals are zero when no faults and differ from zero in the presence of faults. Observers are responsible for generating residuals using the mean square error. The error will be responsible for deciding when system failure occurs and what type of failure is affecting the system, as a matter of fact, this consists of the diagnostic stage. The discriminator $\hat{r}$ is an appropriate function based on the estimate error that is capable of taking decisions when there is a failure.

The fault isolation is to identify the $\hat{r}$ discriminator, because there is no fault within the range $0 < \hat{r} < 1$; case that the system should act in nominal terms, the plant is operating normally. Any other option failure is isolated by operating the output estimator system.

5. Case of study and results

5.1 Technical Description

The bottling of the FUAC plant has two sections, one of homogenization (Figure 6) and another for bottling. The homogenization section is a hydraulic coupling of four tanks unpressurized with solenoid valves, manual valves, and pumps arranged to allow fluid to flow between them. The first tank (T1) is the liquid receiving tank that reaches the plant, the second and third tanks (T2-T3) are the cooling and heating tanks, respectively. The fourth tank (T4) is the homogenization tank. This tank has a mixer, one analog ultrasonic level sensor 4 to 20 mA and the RTD. The maximum capacity is 200 liters, and due to the location of the RTD in tank T4, the minimum capacity must be 80 liters. Three of the four pumps have a frequency driver Power Flex 40 Allen-Bradley brand. It also has 6 valves and 11 manual valves. Besides this, it includes three touch screens, one Panel View Plus 1500 and 2 panels View Plus 600 Allen-Bradley.

Figure 6. Homogenization Plant. Source: own.

4.2 Model Identification

The model of the homogenization tank (T4 Figure 7) of permanent agitation appears after a balance of matter and energy (18) - (19).

$$\rho A \frac{dh}{dt} = \rho\left(Q_C(t) + Q_F(t) + Q_A(t)\right) - \rho K\sqrt{h(t)} \quad (18)$$

$$B = Q_C T_C + Q_F T_F + Q_A T_A \rho C_p A \frac{d(h(t)*T(t))}{dt} = \rho C_p[B - K\sqrt{h(t)}T(t)] \quad (19)$$

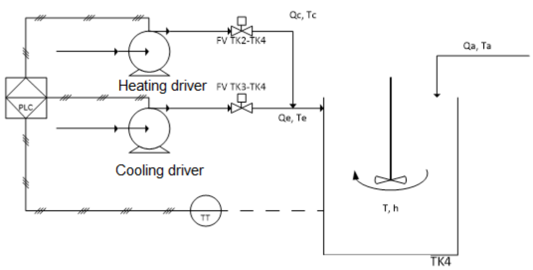

Figure 7. Homogenization process. Source: own.

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Where $h(t)$ and $T(t)$ are the state variables, $h(t)$ in [m] is height of the liquid in T4 and $T(t)$ in [ºC] is temperature of the liquid in the T4. The liquid density ρ is in [kg/m^3], A in [m^2] is the transversal area of the tank, $Q_C(t), Q_F(t), Q_A(t)$ in [m3/s] are flows from hot water, cold water, and the environment respectively. C_p in [J/kgºC] is the heat capacity at constant pressure of the liquid [14-17], T_C and T_F are the water temperature from the heating and cooling tanks, T_A is the ambient temperature and K is adjustment gain constant.

After the linearization of non-linear model (18) and (19) in (20), where ($h_s, T_s, T_{Cs}, T_{Fs}, T_{As}$) are the deviation variables in the operating point. The values of the constants are:

$h_s = 0.245m, T_s = 28^o C, A = 0.33285m^2, T_{Cs} = 40^o C, T_{Fs} = 5^o C, Q_{As} = 0.00045\frac{m^3}{s}, T_{As} = 23^o C, K = 0.0006$

$$\begin{bmatrix} \frac{d[h(t)-h_s]}{dt} \\ \frac{d[T(t)-T_s]}{dt} \end{bmatrix} = \begin{bmatrix} \frac{-K}{2A\sqrt{h_s}} & 0 \\ 0 & \frac{-K}{A\sqrt{h_s}} \end{bmatrix} \begin{bmatrix} h(t)-h_s \\ T(t)-T_s \end{bmatrix} + \begin{bmatrix} \frac{1}{A} & \frac{1}{A} \\ \frac{T_{Fs}-T_s}{Ah_s} & \frac{T_{Cs}-T_s}{Ah_s} \end{bmatrix} \begin{bmatrix} Q_f(t)-Q_{fs} \\ Q_C(t)-Q_{Cs} \end{bmatrix} +$$

$$\begin{bmatrix} \frac{1}{A} & 0 \\ \frac{T_{As}-T_s}{Ah_s} & \frac{Q_{As}}{Ah_s} \end{bmatrix} \begin{bmatrix} Q_A(t)-Q_{As} \\ T_A(t)-T_{As} \end{bmatrix} \quad (20)$$

Although the space state model (20) is controllable but not observable, the left coprime fraction model (21) must be observable. If we take the model (21) it is simply to get to a canonical form number 4 [9] because it has only one output $T(t)$.

$$\hat{G}(D) = \left[D^2 + \frac{3K}{2A\sqrt{h_s}}D + \frac{K^2}{2A^2h_s}\right]^{-1} * \left[\frac{-23}{Ah_s}D + \frac{-23K}{2A^2h_s^{3/2}} \quad \frac{12}{Ah_s}D + \frac{12K}{2A^2h_s^{3/2}}\right] \quad (21)$$

5.3 FDD for the Homogenization Plant

The Figure 8 shows the fault detection and diagnostic system implemented in Simulink for the homogenization Plant 4.The close loop is consisting in a coupling constant equal to 30. The frequency driver responds to the equation (22) and the linear system of the homogenization tank (20). The residual generation block has the estimators that deliver the output estimated. The difference between the output of temperature $T(t)$ and the output estimated $T_e(t)$ generate the mean square error MSE (see Table 1 for the different faults).

$$\frac{Q(f)}{A} = -7e^{-12}f^3 + 4e^{-9}f^2 + 2e^{-6}f - 7e^{-5} \quad (22)$$

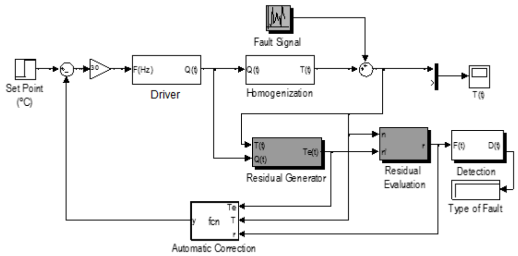

Figure 8. Homogenization plant with the FDD system. Source: own.

The MSE performance index of estimators are in Table 1. When the feedback system has no faults, all estimators have very low MSE, where the two with less error rate are the estimator Luenberger and Lyapunov, this is due to the gain in both cases, it is the same for both using different algorithms if failure is normally distributed randomly. Observers in coprime fraction have the least MSE, followed by observer Lyapunov. However, because of the multivariable nature of the plant, the observer matrixes are not unique [11]; this implies that the results may differ slightly.

Estimator	No Fault	Fault 1	Fault2	Fault3	Fault4
Kalman	5.4257x10-8	3.7584	2.5266	10.0263	-0.010
Luenberg	4.2297x10-31	3.6310	2.1239	9.5293	-0.258
LQ	3.5226x10-7	3.8034	2.4765	10.1852	-0.023
Cyclic	1.1941x10-7	3.7514	2.5281	10.0052	-0.006
Lyapunov	4.2297x10-31	3.6310	2.1239	9.5293	-0.082
C.F.	4.7875x10-8	3.7928	2.5099	10.1207	-0.013

Table 1. MSE for different estimators in continuous time. Source: own.

The residual evaluation block contains the average of the mean square error. Block detection failure in Figure 8 should simply discriminate according to different residues obtained within the block of evaluation. It means detects the fault. Automatic correction block performs fault isolation. If the residue appears into the range $0 < \hat{r} < 1$, (see Figure 9), the block operates with the nominal temperature signal, and if there is a fault: the correction block operates with the estimated temperature signal.

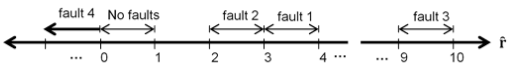

Figure 9. Residual assignation for the different faults. Source: own.

The second index to evaluate the estimators is the elapsed time. Table 2 shows the elapsed time in estimation, after several tests the estimator with lower average time is Luenberger, followed by Lyapunov, and then with similar results are coprime fractions with LQ estimators (see Figure 10). The cyclic design is the longest. In fact, the cyclic design must find any vectors, such that none singularity appears in the solution of the final matrix.

Estimator	No Fault	Fault 1	Fault2	Fault3	Fault4
Kalman	0.8987	1.0468	1.0825	0.6262	0.6415
Luenberg	0.4000	0.4182	0.3822	0.5034	0.6711
LQ	0.6704	0.7045	0.7311	0.4454	0.4134
Cyclic	1.1721	2.6841	2.8604	2.3663	2.9162
Lyapunov	0.4063	0.4171	0.4221	0.4531	0.4231
C.F.	0.6347	0.7533	0.8086	0.7793	0.5475

Table 2. Elapsed time in the estimation in seconds. Source: own.

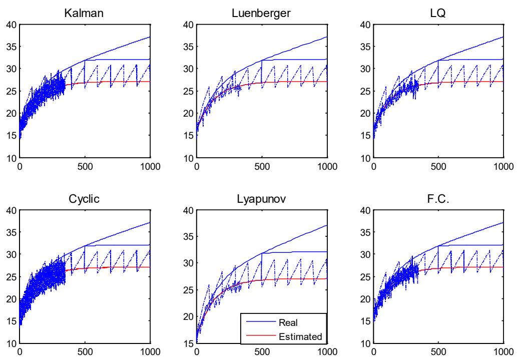

Figure 10. Estimation with the different faults. Source: own.

5.4 OPC Communication

The scheme performed in Simulink is shown in Figure 11; there are placed subsystems to capture data using OPC tools. The test system with a coprime fractions estimator is a step fault (disconnection fault on RTD). In the Software Factory Talk View Studio, an interface alarm that alerts the operator when a failure occurs and informs the detected fault which was scheduled. This message is intermittent each 4 seconds. Once the fault is corrected, a new message appears, informing that the fault has been corrected using the estimator.

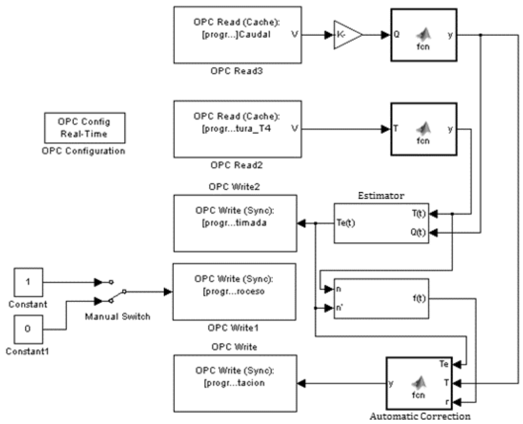

Figure 11. OPC Communication. Source: own.

6. Conclusions

Methods based on state estimators and observers are most often applied in the field of fault detection largely due to the implementation. According to the elapsed time, the Luenberger and Lyapunov are the fastest; in some cases, the results of both estimators are similar. Jordan estimator is the slower because of the loops that need it per eigenvalue when the Jordan canonical form is not cyclic. This estimator also performs a good low error in estimation, almost similar to the coprime fraction design and the LQ estimator. Though for multivariable systems the observer matrixes are not unique, the coprime fraction estimator ends in an observable or controllable canonical form, for that reason the results of coprime fractions and canonical form estimators are equals, and both are taken as one.

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This is appropriate because now there are no tools can simulate systems in matrix polynomial fractions models.

The assignation of faults for the detection and diagnostics system depends on the residuals over the mean square error. This means that the diagnostic system could change according to the residuals positions, the idea is per faults the system must be discriminating residuals based on some kind of operation; in our case, we use the average of mean square error. The isolation of additive faults is simple when the fault is not very small like the situation where the output was without faults. Probably in that case, the FDD system will be incapable of perceiving the faults.

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