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<https://doi.org/10.14483/issn.2248-4728>



A CASE-STUDY VISION

### Modeling and implementation of a thermal plant

*Modelamiento e implementación de una planta térmica*

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#### Abstract

This paper provides the mathematical modeling and implementation of a thermal plant. In order to do this, the equations of a thermal system are deduced through heat relations applied to increase temperature, and the heat amount that is lost by different types of heat transfer; likewise, from the passage of these relations to the equations of state - through application of Laplace transform -, the system's transfer function is obtained, which allows a comfortable implementation in software and the respective simulation where control actions can be established.

Finally, a prototype of a thermal plant is shown, consisting of a thermal box divided into two sections: one that has a double filament bulb as a heating element, and another with two fans

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used for heat dissipation. The circuits and materials for the actuators, sensors and power supplies are designed and implemented. In this kind of thermal system, the input is a voltage signal that manages the temperature in both box chambers; one of the outputs is another voltage signal that proportionally shows the chamber temperature where the heat source is, the other is a voltage proportional to the temperature of the other chamber that has a thermal exchange with the first one by means of a separating membrane; that is, for industrial and didactic purposes not developed in the document, the thermal exchange between chambers using sensors and actuators can be shown.

**Keywords:** Actuator, Heat, Sensor, Source, Temperature, Thermal plant, Transfer function.

## Resumen

En este artículo se presenta el modelamiento matemático e implementación de una planta térmica. Para ello, se deducen las ecuaciones de un sistema térmico a través de la relación de calor aplicado para incrementar la temperatura, y la cantidad de calor que se pierde por los diferentes tipos de transferencia térmica; así mismo, del paso de estas relaciones a las ecuaciones de estado - por medio de la aplicación de la transformada Laplace -, se obtiene la función de transferencia del sistema que permite una cómoda implementación en software y la respectiva simulación donde pueden establecerse acciones de control. Se expone, finalmente, la implementación de un prototipo de planta térmica, el cual consta de una caja térmica dividida en dos secciones: una que tiene como elemento calefactor un bombillo de doble filamento, y otra donde dos ventiladores se utilizan para la disipación de calor. Se diseñan e implementan los circuitos y materiales para los actuadores, los sensores y las fuentes de alimentación. En tal sistema térmico, la entrada es una señal de voltaje que maneja la temperatura en las dos cámaras de la caja; una de las salidas es otra señal de voltaje que con alguna proporción muestra la temperatura en la cámara donde está la fuente de calor, la otra es un voltaje

proporcional a la temperatura de la otra cámara que tiene un intercambio térmico con la primera por medio de una membrana separadora; es decir: puede evidenciarse, con fines industriales y didácticos no desarrollados en el documento, el intercambio térmico entre cámaras usando sensores y actuadores.

**Palabras clave:** Actuador, Calor, Sensor, Fuente, Temperatura, Planta térmica, Función de transferencia.

## 1. Introduction

The study of thermal systems becomes relevant in the literature given the multiple applications in industrial processes [1], [2]. Linearity, speed, downtime and interference are, among others, the most outstanding characteristics for analysis [3-5].

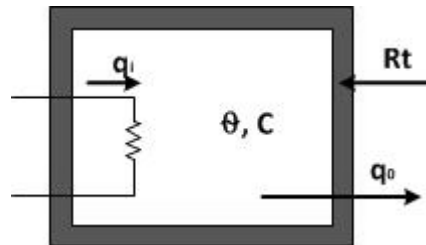
On the other hand, regarding its design, there is a large number of methodologies, for example, in [6] primary models are proposed for optimization; and in [7-11] principles for simulation are proposed: to calculate values of operating variables (pressures, temperatures, mass flow and energy for particular thermodynamic states), in order to predict behavior in specific applications. In this document, it is proposed a mathematical modeling based on heat transfer and dissipation [12], [13], [14], that implicitly considers homogenization in the effective properties of the heat equation, [15].

The contribution, from the given considerations, of the present research is to propose a prototype implementation of a thermal box -with actuator and sensor circuits, using operational amplifiers and transistors- [16], [17], as well as the required power supplies for the operation. The above as an alternative to develop prediction and optimization in applications of academic and industrial environment.

## 2. Modeling of a thermal plant

With a systemic approach to industrial processes, an analysis of the dynamics of this system will be carried out.

**Figure 1.** Thermal System with a Thermal capacity.



Source: own.

From figure 1 it can be seen that in thermal systems there is: a steady state that can be related between the heat amount that makes the system's internal temperature increase; and the heat amount that is lost by several types of heat transfer, inherent to the thermal system itself. Therefore, the relationship between temperature increase and heat flow increase is given by (1) y (2):

$$\hat{\theta}(t) = \theta(t) - \bar{\theta} \quad (1)$$

$$\hat{q}(t) = q(t) - \bar{q} \quad (2)$$

$\hat{\theta}(t)$  : Temperature variation

$\theta(t)$ : Initial temperature

$\bar{\theta}$  : Lost temperature

$\hat{q}(t)$ : Heat flow variation

$q(t)$ : Initial heat flow

$\bar{q}$ : Lost heat

Where  $\theta$ , temperature in Kelvin ( $^{\circ}\text{K}$ );  $q$ , heat flow in Joule per second (J/s) or Watts (W).

When values are nominal for  $\bar{\theta}$  y  $\bar{q}$ .

The thermal capacity of this system is the relationship between the temperature of the thermal body and the heat it stores. This relationship is considered linear.

If it denotes the net heat flow between the system body at any time as:  $q_{in}(\lambda) - q_{out}(\lambda)$ , where the net heat applied is between  $t_o$  y  $t$ , then it has (3).

$$\int_{t_o}^t [q_{in}(\lambda) - q_{out}(\lambda)] d\lambda \quad (3)$$

If it is assumed that the heat applied over a time period is constant and it is the temperature that changes  $\theta$ ; the initial temperature in zero time  $t_o$  will be given by  $\theta(t_o)$ , in (4).

$$\theta(t) = \theta(t_o) + \frac{1}{c} \int_{t_o}^t [q_{in}(\lambda) - q_{out}(\lambda)] d\lambda \quad (4)$$

Constant  $c$  is known as the thermal capacity, given in Joules per Kelvin, if a body is considered to have a known amount of mass  $M$  and a specific heat  $\sigma$ .

Deriving (4) gives (5);

$$\dot{\theta} = \frac{1}{c} [q_{in}(t) - q_{out}(t)] \quad (5)$$

In the given thermal system, heat has three ways to spread from one spot to another: conduction, convection and radiation. In this case, only conduction is taken into account; consequently, there will be two different temperatures between the starting point of conduction and the end point of conduction. Then the relationship between the initial temperatura  $\theta_1$  and the final temperature  $\theta_2$  will be:

$$q(t) = \frac{1}{R} [\theta_1(t) - \theta_2(t)] \quad (6)$$

Pointing out that  $R$  Pointing out that it is the thermal resistance of the conduction medium between these two spots; and the units of this ratio will be kelvin-second per joule, or kelvin per watt (K/W), and is given by (7):

$$R = \frac{d}{A\sigma} \quad (7)$$

Where  $d$  is a distance between the body spots and  $A$  is the body cross-sectional area and  $\sigma$  is the already mentioned thermal conductivity.

Performing an analysis on Figure 1. There are (8) and (9):

$$q_{in}(t) = q_i(t) \quad (8)$$

$$q_{out}(t) = q_o(t) \quad (9)$$

Clearing from (6),

$$q_o(t) = \frac{1}{Rt}(\theta_i - \theta_a) \quad (10)$$

Based on the above, the state variable (11) is obtained:

$$\dot{\theta} = \frac{1}{c} \left[ q_i(t) - \frac{1}{Rt}(\theta_i - \theta_a) \right] \quad (11)$$

As outside body temperature is proportional to room temperature, and the heat source is represented by  $q_i(t)$ , clearing (11):

$$\dot{\theta} + \frac{1}{RtC} \theta_i = \frac{1}{c} q_i(t) + \frac{1}{RtC} \theta_a \quad (12)$$

By updating this first-order linear differential equation for a constant time given by  $\tau = RC$  and the data inputs  $q_i$  and  $\theta_o$ , it is obtained (13):

$$\frac{1}{RtC} \bar{\theta}_i = \frac{1}{c} \bar{q}_i + \frac{1}{RtC} \theta_a \quad (13)$$

As the system is balanced for a given time, thermal capacity remains constant and heat flow in and out of the thermal resistance is equal to the heat output, then it gives (14):

$$\bar{\theta}_i = \theta_a + Rt\bar{q}_i \quad (14)$$

And it is defined by (15):

$$\hat{\theta}(t) = \theta_a(t) - \bar{\theta}_i \quad (15)$$

$$\hat{q}_i(t) = q_i(t) - \bar{q}_i \quad (16)$$

By performing the substitution in (12) it gives (17) as a result:

$$\dot{\hat{\theta}} + \frac{1}{RtC} (\hat{\theta} + \bar{\theta}) = \frac{1}{c} [\hat{q}_i(t) + \bar{q}_i] + \frac{1}{RtC} \theta_a \quad (17)$$

Using the equation (13) and canceling the variables established as constants is obtained (18):

$$\dot{\hat{\theta}} + \frac{1}{RtC} \hat{\theta} = \frac{1}{c} \hat{q}_i(t) \quad (18)$$

Therefore, for the given system the equation of state (11) and output (14) are considered:

$$\frac{d\theta_i}{dt} = \frac{1}{c} \left( q_i - \frac{\theta_i}{Rt} + \frac{\theta_a}{Rt} \right) \quad (19)$$

$$q_o = \frac{\theta_i - \theta_a}{Rt} \quad (20)$$

Now, applying Laplace transform gives (21) and (22):

$$s\theta_i(s) = \frac{1}{c} \left( q_i(s) - \frac{\theta_i(s)}{Rt} + \frac{\theta_a(s)}{Rt} \right) \quad (21)$$

$$q_o = \frac{\theta_i(s) - \theta_a(s)}{Rt} \quad (22)$$

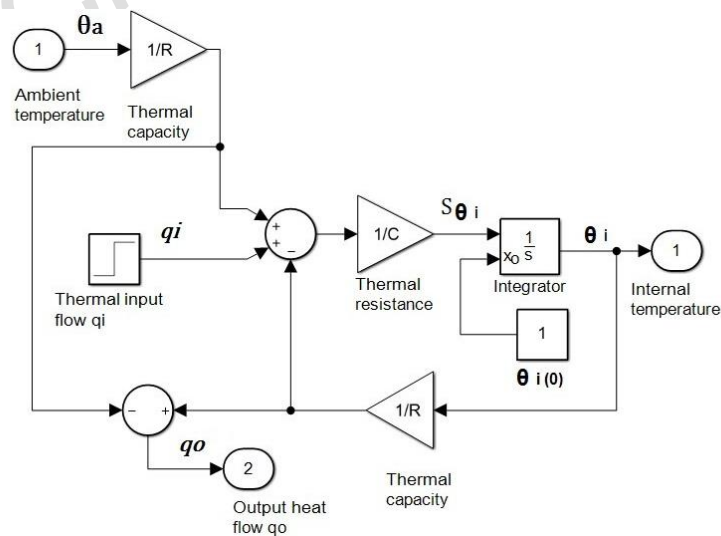
Thus, the transfer function is given by (24):

$$H(s) = \frac{y(s)}{u(s)} \quad (23)$$

$$H(s) = \frac{\frac{1}{c}}{s + \frac{1}{RtC}} \quad (24)$$

On the other hand, the state equation in block diagram by MatLab® Simulink, is shown in Figure 2.

Figure 2. Basic Thermal System.

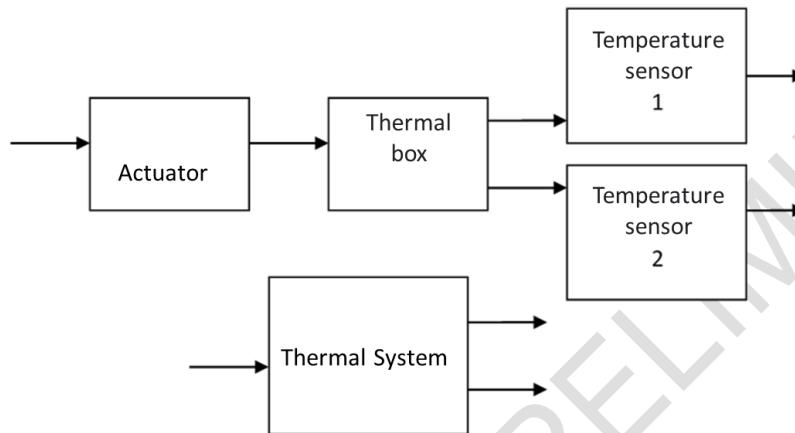


Source: own.

### 3. Thermal plant implementation example

Below is the implementation of a thermal system with its respective actuator and sensors. System overview is given in figure 3.

**Figure 3.** System block diagram.



Source: own.

This system is composed of one input and two outputs. The system input is a voltage signal that manages temperatures in two chambers of a thermal box. One of the system outputs is a voltage signal that with some proportion shows the temperature in a chamber where the heat source is, this corresponds to the temperature sensor 1.

The other output is a voltage that is proportional to the temperature of the second chamber that has a thermal exchange with the first one by means of a separating membrane; this output corresponds to temperature sensor 2.

#### 3.1. Actuator circuit

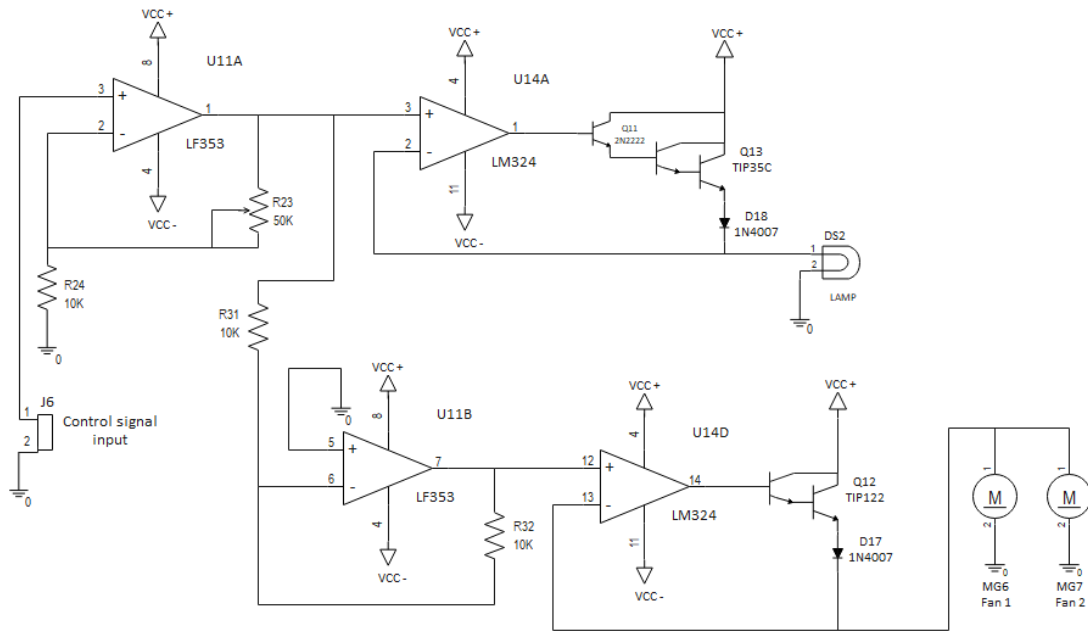
The actuator is a device that gives enough energy to the system so that it can be manipulated. In this case a 12v double filament bulb with a power of 24W and two 12v fans are handled.

The Jumper (J6) indicates an input connection to the system, which should be a voltage varying between +/- 5 volts. From 0v to +5v the bulb is turned on at full power and the fans are disabled, with a voltage range of 0v to -5v the fans are running and the bulb is off.



The actuator circuit is represented in figure 4 and table 1.

**Figure 4.** Thermal system actuator circuit.



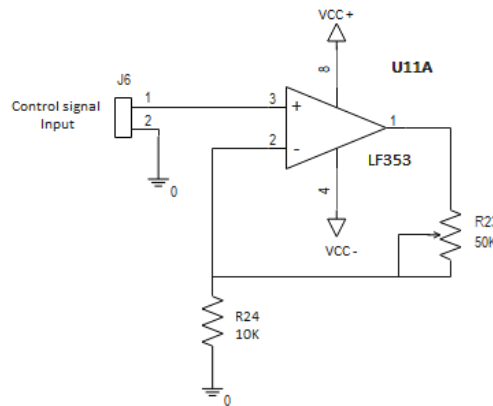
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**Table 1.** Actuator Components

Quantity	Components
1	Power strip
2	LM 324
2	LF353
1	TIP 35C
1	TIP 122
1	2N2222
2	Diodes 1N4007
1	Trimmer 50K
3	Resistances 10K
2	12V Fans
1	24W Light bulb
1	2 inputs terminal block

Source: own.

**Figure 5.** Source signal amplification or calibration stage.

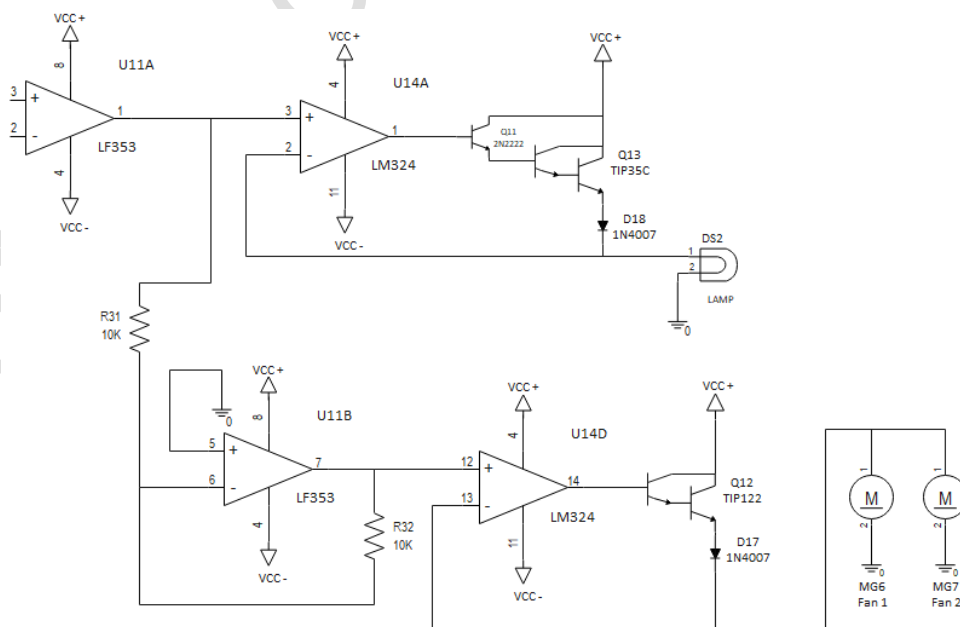


Source: own.

This stage is part of the actuator. Then there is an operational amplifier configured as a non-inverting gain (LF353 U11A) that amplifies the voltage coming through J6. This stage serves as an output signal calibration that acts on the bulb or fans, Figure 5.

The gain of the non-inverting operational amplifier (LF353 U11A) is variable by changing the potentiometer R23 resistance value. It is designed to ensure that when a system input is +/-5v, the light bulb or the fans reach +/- 12v at maximum gain, figure 6.

**Figure 6.** Actuator power stage.



Source: own.



inverting operational amplifier, LM324 U14B. This acts as a variable gain amplifier that has a trimmer R27 adjustment of 100KΩ. This trimmer allows the user to adjust temperature scale, that is, to adjust the output temperature to a voltage of 5v. Jumper J10 is an output connection for this sub-system; it is a voltage signal, table 2.

**Table 2.** Position sensor components.

Quantity	Components
1	LM35
1	LM 324
1	Power strip
1	Trimmer 100KΩ
1	Resistance 10KΩ
1	2 inputs terminal block

Source: own.

### 3.3. Power supply circuit

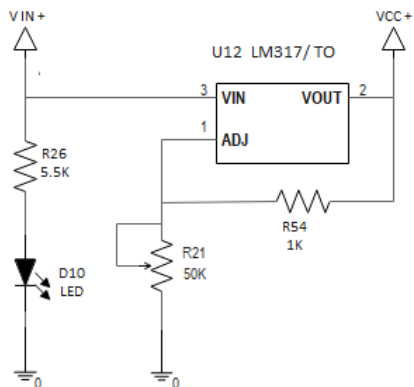
The circuit uses different supply voltages. In general, it is powered by a dual +/-20v source on the circuit inputs VIN+ and VIN-, figure 8-9, table 3.

The Operational Amplifiers are powered with +/-15v. These voltages are generated by the following regulators: LM 317 for positive source +15v; and LM 337 for negative source -15v.

Potentiometers (R21, R22) connected to each of the regulators allows setting an exact output voltage for each setup.

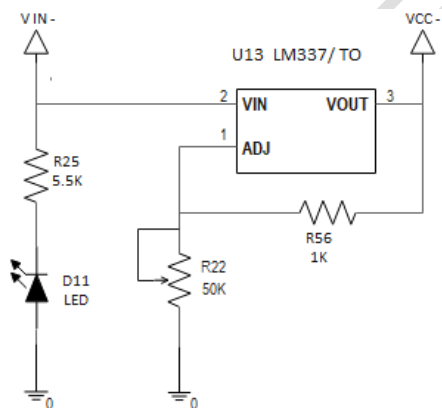
The light-emitting diodes on the voltage regulators (LM317 and LM337) are used to recognize that the external source is connected.

**Figure 8.** 15 Volt power supply.



Source: own.

**Figure 9.** -15 Volts power supply.



Source: own.

**Table 3.** Power supply circuit components

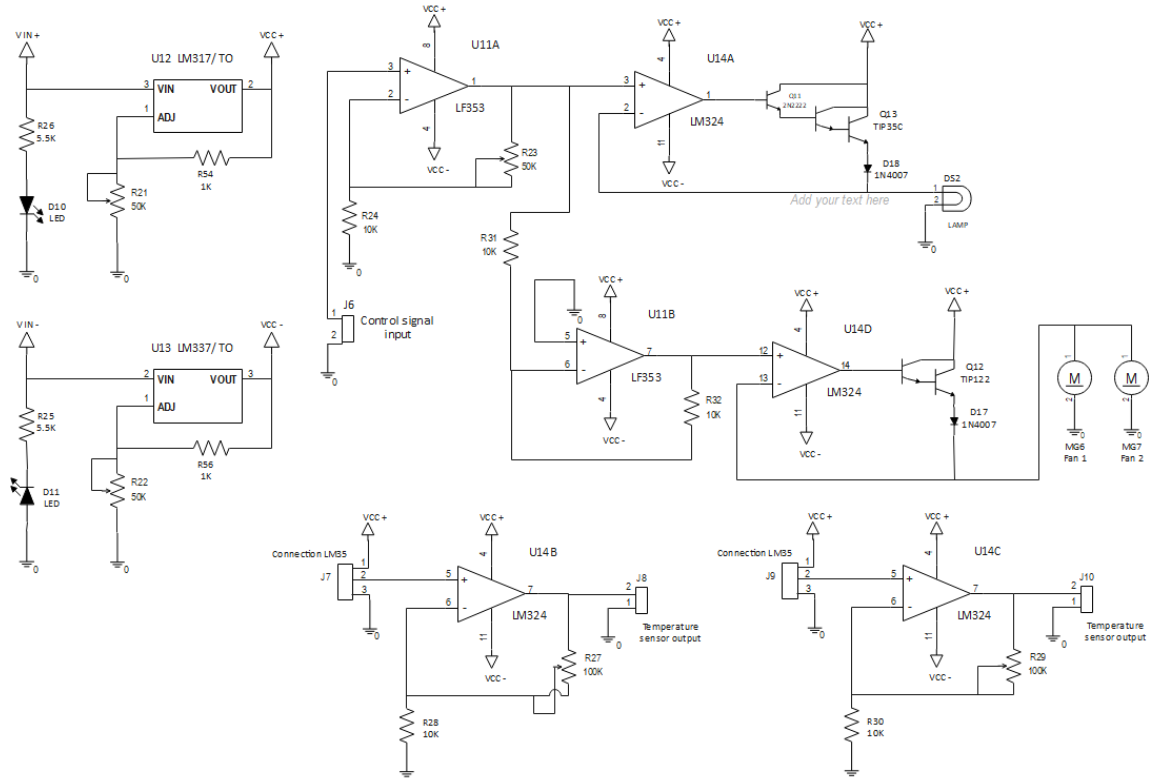
Quantity	Components
1	Power strip
1	LM 337
1	LM 317
2	Trimmer 50K
4	Resistances (2)1K,(2) 5.5K
2	Leds
1	3 inputs terminal block

Source: own.

### 3.4. Overall assembly schematic

In Figure 10 is presented the overall assembly schematic.

Figure 10. Overall Thermal System schematic.

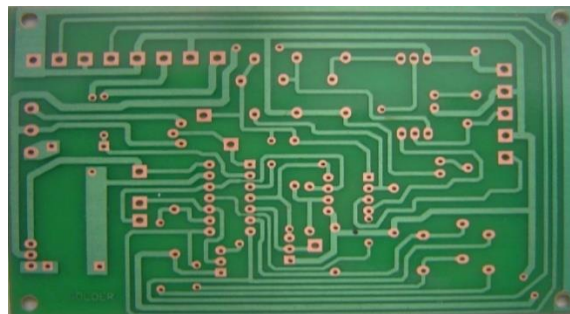


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### 3.5. Printed Circuit Board (PCB)

The printed circuit board is presented in Figure 11.

Figure 11. Bottom view.

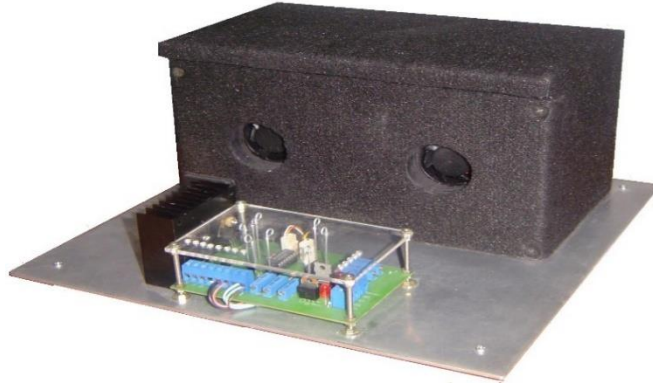


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### 3.6. Finished prototype

The finished prototype is shown in figures 12-18.

**Figure 12.** Thermal plant.



Source: own.

**Figure 13.** Thermal plant.



Source: own.

**Figure 14.** Thermal plant.



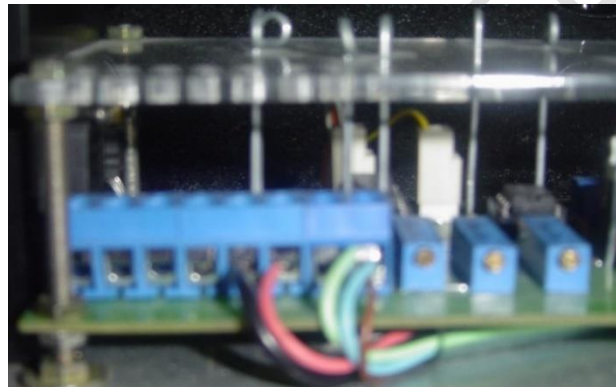
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**Figure 15.** Thermal Circuit top view.



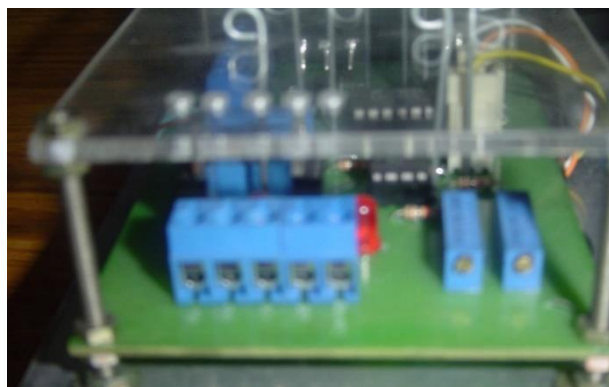
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**Figure 16.** Thermal System Outputs.



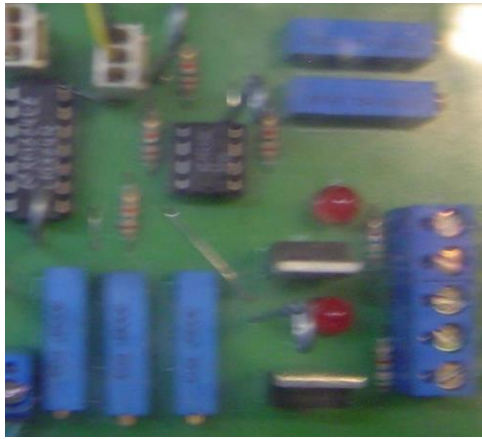
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**Figure 17.** Thermal System Inputs.



Source: own.



**Figure 18.** Adjustment trimmers.

Source: own.

#### 4. Conclusions

Mathematical modeling development, besides making the considered phenomenon comprehensible, generated state equations that were expressed and set up in a block diagram in MatLab® Simulink for simulation and later application of some type of controller, actions that can be determined by the user according to each corresponding application.

The linearity and accuracy of the LM35 sensor make it easy to measure temperature inside chambers.

The actuator circuit works properly and effectively in turning fans and light bulbs on and off. There is a small noise sensitivity.

Heat transfer and dissipation are slow processes that consequently slow down the dynamics and behavior of the thermal plant.

The applications have this kind of work as starting points, however its depth and complexity, currently, are based on concepts such as Thermoeconomics discipline that merges concepts of thermodynamics and root economics for new predictive research and optimization for applications of academic and industrial environment.

## Acknowledgements

The authors would like to thank the research groups SciBas and ORCA, attached to the Center for Research and Scientific

Development of the Universidad Distrital Francisco José de Caldas, for the support and advice received for developing this research.

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