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A RESEARCH VISION

Design and implementation of LQG controller in Ball & Beam system

Diseño e implementación de controlador LQG en sistema Ball & Beam

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ABSTRACT

The Ball & Beam system is one of the most complete case studies in control engineering, because it is a non-linear and naturally unstable system. In this article we propose to make an optimal LQG controller for Quanser's Ball & Beam system, composed of a linear quadratic regulator (LQR) and a linear quadratic estimator (Kalman filter) with which the noise of the system's ball position signal was eliminated, managing to mitigate the problems generated by the high sensitivity to sensor noise. Starting from the state space representation of the Quanser Ball & Beam system and using the Matlab/Simulink software and its QUARC module, an optimal LQG controller was designed, simulated and implemented in the Quanser Ball & Beam system. Finally, the simulation results and implementation show that the LQG controller is effective in controlling the Ball & Beam system despite the noise presented by the feedback signal.

RESUMEN

El sistema Ball & Beam es uno de los casos de estudio más completos dentro la ingeniería de control, debido a que es un sistema no lineal y naturalmente inestable. En el presente artículo se propone realizar un controlador optimo LQG para el sistema Ball & Beam de Quanser, compuesto por un regulador cuadrático lineal (LQR) y un estimador cuadrático lineal (filtro de Kalman) por medio del cual se estimaron todos los estados del sistema, logrando mitigar las problemáticas generadas por la alta sensibilidad al ruido del sensor. Partiendo de la representación en espacio de estados del sistema Ball & Beam de Quanser y utilizando el software Matlab/Simulink y su módulo QUARC se diseñó, simulo e implemento un controlador optimo LQG en el sistema Ball & Beam de Quanser. Finalmente, los resultados de la simulación e implementación muestran que el controlador LQG es efectivo para controlar el sistema Ball & Beam a pesar del ruido que presenta la señal de realimentación.

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1. Introduction

In the process of controlling dynamic systems, it is common to find several complications due to the instability of the systems, the presence of nonlinear components or the high sensitivity to noise of the measuring instruments that integrate them. The Ball & Beam system is one of the most complete case studies, since it presents all the mentioned complications, and whose objective is to stabilize the position of a ball around an operating point, by means of the variation of the inclination angle of a beam. Within the academy it is common to find several authors who have developed and applied different control techniques for this system, some of the documented cases will be described below.

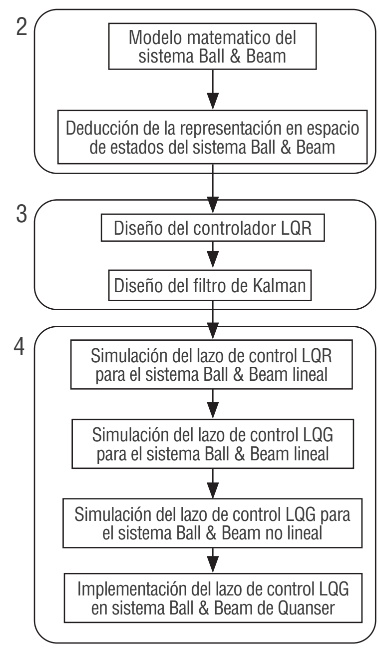
With the objective of contemplating all the dynamics of the system, the nonlinearity presented by the friction between the ball and the beam has been included in the mathematical modeling of the Ball & Beam system [1]. The direct Lyapunov method has also been used to describe the unstable behavior of the system [2]. Along with this, classical control loops were developed such as PID controllers in series or in cascade [3] proportional feedback in state space and Lag/Lead compensators, which, since they do not have any criteria for the control of the system, have been developed. [2] which, not having optimal design criteria, generate inefficiency and limit the operating conditions of the system.

These drawbacks generated those different authors raised various solutions from modern control theories, such as optimal controllers which improved the efficiency of the system. The optimal controller most named in the literature is the linear quadratic controller, which from weight matrices established both heuristically and by optimization methods, minimizes the energy required to control the system [4-5]. Parallel to this, quasi-optimal control techniques were developed in time, which start from the transformation of the system to the Jordan model [6]

On the other hand, it is common that the signals of the variables to be controlled present noise components that directly affect the performance of the controller, for this reason the LQG controller was developed alternatively [5] was developed, where a Kalman filter is used, constructed from the state equations of the system or using the nonlinear differential system written in Brunovsky's canonical form [4]. Through this filter the system states are estimated, eliminating the noise of the associated signals and improving the system performance. In addition to this, variants of this control technique were developed, such as the robust LQG/LTR controller, which presents a better performance against disturbances or uncertainties that may occur in systems such as Ball & Beam, or in rotary positioning systems [1] or in direct drive rotary positioning systems [7].

The particular characteristics of the LQG controller have generated that it is used in external control loops for the Ball & Plate system [8] system, and as a control strategy for much more complex systems such as the Ballbot robot. [9]. Even this control technique has been integrated with the feedforward controller for the development of control loops for NASA network antennas [10].

Figure 1. Methodology used for research development



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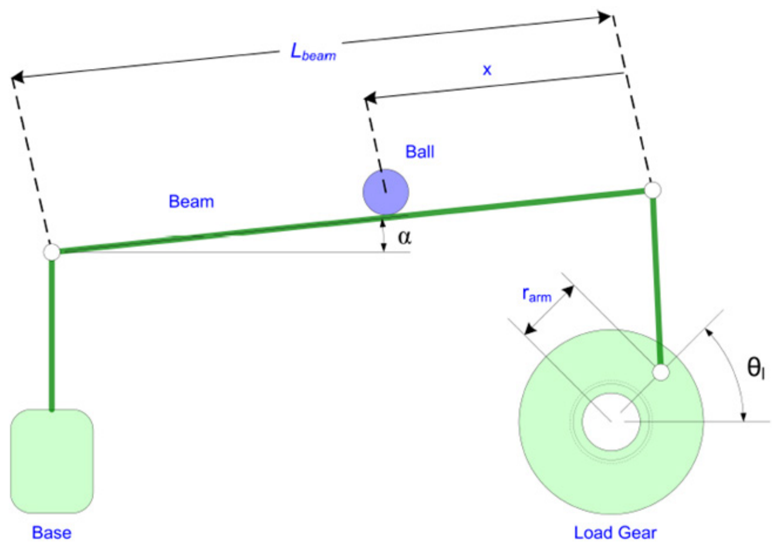
The focus of this work is to carry out the design, simulation and implementation of the LQG control technique for the Quanser Ball & Beam system, in order to validate its operation in a nonlinear dynamic system, naturally unstable and also has high sensitivity to noise in their measuring instruments. In this order of ideas, the methodology used in this work is presented in the following sections as follows: section II emphasizes the dynamic model of the system, then in section III, the linear model of the Ball & Beam system will be taken and the design of the LQR, LQG and Kalman filter controller will be detailed; in section IV the simulation of the different control loops will be developed, and the results obtained in the implementation of the LQG control loop in the Ball & Beam system of Quanser will be shown. Figure (1) shows in more detail the development of each of the above sections.

2. Mathematical model of the Ball & Beam system

To develop a model-based controller it is necessary to obtain a mathematical model that describes the behavior of the system over time, this model can be obtained both from physical laws and from the system's own measurements. As mentioned in the previous section, the platform to be analyzed is the Quanser Ball & Beam system, which is composed of two subsystems, the first one describing the dynamics of the servomotor and the second one describing the dynamics between the servomotor and the servo motor. and the second one that comprises the dynamics between the ball and beam. . Figure 2 shows a general diagram of Quanser's Ball & Beam system, where the lengths and relevant variables of the system are defined, which were taken into account within its mathematical model.

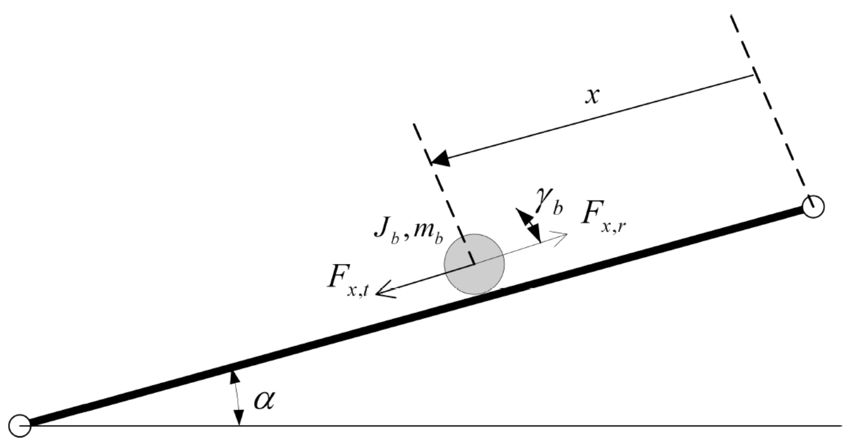
Figure 2. General diagram of the Quanser Ball & Beam system [11].



2.1. Mathematical model

The dynamic model of the ball and beam is derived from Newton's second law of motion. [11]. Taking as a starting point the free body diagram of the subsystem, shown in Figure 3, the forces acting on the ball are deduced, where $F_{x,t}$ is the translational force generated by gravity and the force $F_{x,r}$ is the inertia of the ball.

Figure 3. Free body diagram of the ball girder subsystem. [11]



Starting from equation (1) given by Newton's second law, equation (2) related to the nonlinear dynamic model, which describes the motion of the ball along the beam, is derived; the parameters used for the modeling of the system are shown in Table 1.

$$m_b\left(\frac{d^2}{dt^2}x(t)\right) = F_{x,t} - F_{x,r} \tag{1}$$

$$\frac{d^2}{dt^2}x(t) = \frac{m_b g \sin\alpha(t) r_b^2}{m_b r_b^2 + J_b} \tag{2}$$

On the other hand, for the servomotor subsystem, the identification made in Quanser's Workbook SRV2 was taken up again [11]. This transfer function is given in equation (3) and describes the behavior of the servomotor with no load and with the highest gear configuration.

$$P_s(s) = \frac{\theta_l(s)}{V_m(s)} = \frac{1{,}53}{s\,(0.0248\,s + 1\,)} \tag{3}$$

Finally, by finding the relationship between the beam inclination angle α and the angular position of the gear θ_1, equation (4) is obtained, which is linearized, defining that for angular variations of θ_1, close to 0°, it can be approximated that $sin(\theta_1) \approx \theta_1$.

$$\frac{d^2}{dt^2}x(t) = \frac{m_b g \sin\theta_l(t) r_{arm} r_b^2}{L_{beam}(m_b r_b^2 + J_b)} \tag{4}$$

$$\frac{d^2}{dt^2}x(t) = \frac{m_b g \theta_l(t) r_{arm} r_b^2}{L_{beam}(m_b r_b^2 + J_b)} \tag{5}$$

Table 1. Physical parameters of the Quanser Ball & Beam system.

Parameter	Und.	Description
L_{beam} =42.55	*cm*	Beam length
r_{arm}= 2.54	*cm*	Distance between the servomotor gearing and the coupled joint
r_b= 1.27	*cm*	Ball radius
m_b= 0.064	*Kg*	Ball mass
g= 9.8	m/s^2	Gravitational acceleration
J_b=0.000004129	Kgm^2	Moment of inertia of the ball

Source: own.

2.2. *State-space representation of Quanser's Ball & Beam system*

For the design of the LQG controller it is necessary to have a state space representation of the system; for this reason, the anti-Laplace transform was applied to equation (3), and the constants of equation (5) were simplified, obtaining the two differential equations of the same order, which were the starting point to define the state vector of the system, which is composed of the positions and linear and angular velocities $x(t) = [x_p; \frac{dx_p}{dt}; \theta_l; \frac{d\theta_l}{dt}]$. Next, equations (6) and (7) show the state space representation of Quanser's Ball & Beam system.

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0.4179 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -40.32 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 61.69 \end{bmatrix} Vm \tag{6}$$

$$y = [\,1 \;\; 0 \;\; 0 \;\; 0\,]\, x(t) \tag{7}$$

3. LQG controller design

The LQG controller is composed of a linear quadratic regulator (LQR) and a linear quadratic estimator or Kalman filter; this section presents the design of these two subsystems, using the state space representation of the linear system, to later validate the proposed methodology in the nonlinear system.

3.1. *LQR Control*

The main objective in this control loop is to find a feedback matrix *K* that minimizes the energy performance index *J* [5] given in the cost function described in equation (8), where and *ϱ* are symmetrical weight matrices *R*, defined for this research according to the degree of penalty to be established for the states and inputs of the system, thereby minimizing the energy used by the controller and by each of the states.

$$J = \int_0^\infty (\,x^T Qx + u^T Ru)dt \tag{8}$$

To obtain the matrix *K*, an analysis of the problem is made from the Euler-Lagrange equations and application of the Variational Calculus, resulting in the control law in the form of state feedback given in equation (9).

$$K = -R^{-1} B^T P \tag{9}$$

Where P is the algebraic solution of the Riccati equation, described in equation (10).

$$A^T P + PA - PBR^{-1} B^T P + Q = 0 \tag{10}$$

Finally, an optimal state feedback was performed according to the control law $u = -Kx$, where is K the optimal gain matrix found previously.

3.2. Kalman Filter

The Kalman filter is widely used in fields such as digital image processing, computer vision, pattern recognition and state estimation for stochastic systems.

The Ball & Beam system has a variable resistance sensor that allows measuring the position of the ball along the beam, however, the measurement of this state is affected by the high sensitivity to noise that the measuring instrument presents, causing the measurement of the state to x_p become a stochastic process. Therefore, it was necessary to define a state space representation of the system, where the components $w(t)$ associated to the process noise and $v(t)$ associated to the noise present in the measurement of the output variable were added to the dynamics of this state, as shown in equations (10) and (11).

$$\dot{x}(t) = Ax(t) + Bu(t) + w(t) \tag{10}$$

$$y(t) = Cx(t) + v(t) \tag{11}$$

Then, since the analysis, design and implementation of the Kalman filter is simplified by using the dynamic equations of the system in the discrete domain, it was decided to use a zero-order retainer to discretize the system given by equations (10) and (11); obtaining as a result the system described in equations (12) and (13), where the matrices A_d, and B_d are wd related to the equation of states of the system in the discrete domain, and the matrices C_d and vd are related to the output equation of the system in the discrete domain.

$$x_{(k+1)} = A_d x_{(k)} + B_d u_{(k)} + wd_{(k)} \tag{12}$$

$$y_{(k)} = C_d x_{(k)} + vd_{(k)} \tag{13}$$

Based on the above, the Kalman filter was designed, which is an algorithm derived from the optimal state estimation, where it is stated that the random components and $wd_{(k)}$ $vd_{(k)}$ have a Gaussian distribution with zero mean and non-zero covariance. Therefore, for this research the matrices Q_E associated to the process noise covariance and R_M associated to the measurement noise covariance were defined.

$$vd_{(k)} \sim N(0, R_M) \tag{12}$$

$$wd_{(k)} \sim N(0, Q_E) \tag{13}$$

Then, as a first step, the prediction and covariance equations were established, which are stated in equations (14) and (15) respectively.

$$\hat{x}_k^- = A_d \hat{x}_{(k-1)} + B_d u_{(k-1)} \tag{14}$$

$$P_k^- = A_d P_{(k-1)} A_d^T + Q_E \tag{15}$$

Secondly, the update equations for both the prediction and the error covariance, described in equations (17) and (18), were declared.

$$k_{(k)} = P_{(k)}^- C_d^T (C_d P_{(k)}^- C_d^T + R_M)^{-1} \tag{16}$$

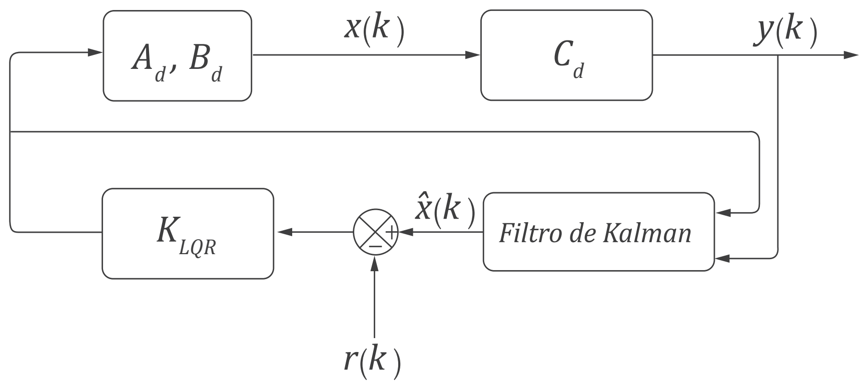
$$\hat{x}_{(k-1)} = \hat{x}_{(k)}^- + k_{(k)}(y_{(k)} - C_d \hat{x}_{(t)}) \tag{17}$$

$$P_{(k)} = (1 - k_{(k)} C_d) P_{(k)}^- \tag{18}$$

The term $k_{(k)}$, described in equation (16), is the optimal gain matrix of the Kalman filter, which minimizes the covariance of the updated error $P_{(k)}$.

Finally, integrating the Kalman filter within the LQR control loop, the LQG control technique is designed, where the cost function for the Ball & Beam system, given by equations (12) and (13), is minimized. Figure 4 shows the interconnection between the Ball & Beam system, the Kalman filter and the set of optimal gains within K the LQG control loop in the discrete domain.

Figure 4. General connection diagram for the LQG control loop.



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4. Simulation and Results

This section shows the design and simulation process of the LQR and LQG controllers in the linear Ball & Beam system, then the simulation of the LQG controller within the nonlinear system is presented; finally, the results obtained in the implementation of the LQG controller in the Quanser Ball & Beam system are shown, a process that was carried out by means of the Matlab/Simulink software and its QUARC module.

Before starting the design process of the LQG and LQR controllers, it was validated that the linear system was both observable and controllable, corroborating that the range of the controllability and observability matrices was equal to the degree of the system. After that, the design of the LQR controller was carried out, where the weight matrices Q and R were configured, as shown below.

$$Q = \begin{bmatrix} 100 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 \\ 0 & 0 & 100 & 0 \\ 0 & 0 & 0 & 10 \end{bmatrix};\ R = [10]$$

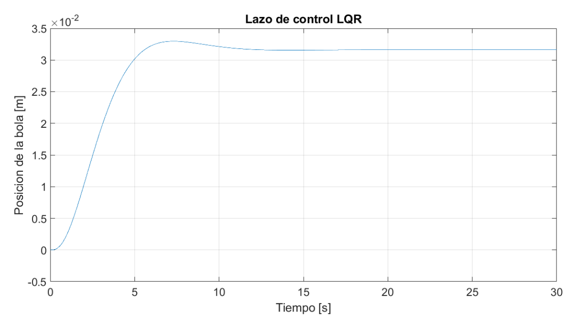
By solving the algebraic Riccati equation, the optimal K feedback gain matrix, described below, was obtained.

$$K = [3.1623\ \ 8.1243\ \ 4.2963\ \ 0.5980]$$

Then the optimal state feedback for the LQR controller was generated in Simulink, where the response of the linear Ball & Beam system was obtained, without noise component in its output state X_p, before a step input of magnitude 0.1. The results of this simulation are shown in Figure 5 , under which it was determined that the system response for this control loop has a settling time of 9.64 sec and a steady state error of 68.38%.

Since the system with the LQR controller presented a high error in steady state, even without integrating the noise component in the optimal state feedback, it was decided to design and integrate a Kalman filter to the control loop, thus forming the LQG control technique for the Ball & Beam system, including with this an integral action to the dynamics of the system, achieving that the response of this to a step input had an error in steady state equal to 0.

Figure 5. LQR control loop, ball position control.



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For the design of the Kalman filter, the covariance matrices and Q_E R_M, associated with the covariance of the process noise and the covariance of the noise of the system output equation, respectively, were declared.

$$Q_E = \begin{bmatrix} 0.1 & 0 & 0 & 0 \\ 0 & 0.1 & 0 & 0 \\ 0 & 0 & 0.1 & 0 \\ 0 & 0 & 0 & 0.1 \end{bmatrix}; R_M = 0.01$$

Using the Kalman filter algorithm, the matrix k, which corresponds to the set of optimal gains of the linear quadratic estimator, was calculated.

$$k = [\ 1.1756 \quad 0.6410 \quad 0.3163 \quad 0.0000]^T$$

For the development of the simulation of the LQG control loop in the linear Ball & Beam system, a random signal of Gaussian distribution with mean 0 and covariance of 0.01 was added in the state feedback x_p, in order to replicate the high sensitivity to noise presented by the linear position sensor of the system. The figure 6 shows the response of the LQG control loop in the linear system and the measurement of the state feedback x_p behavior, once the noise component described above has been added.

Figure 6 (a). LQG control loop for linear system, ball position control.

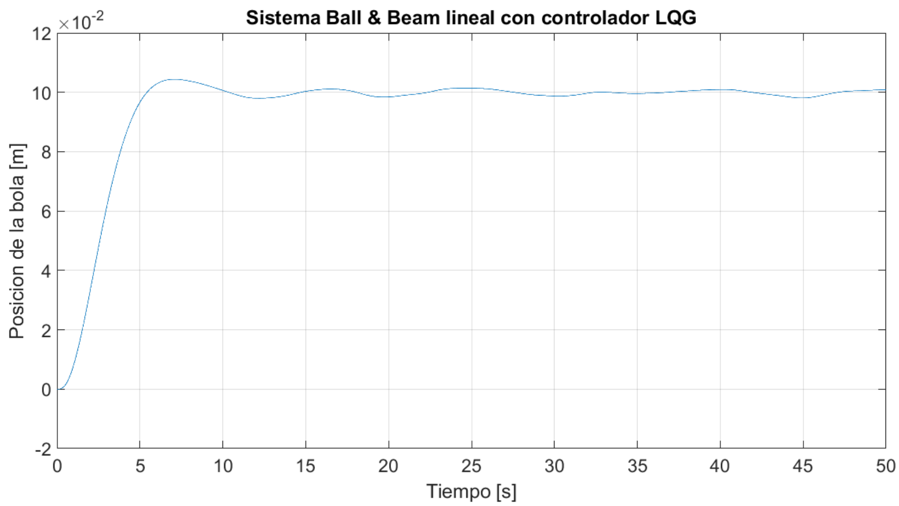

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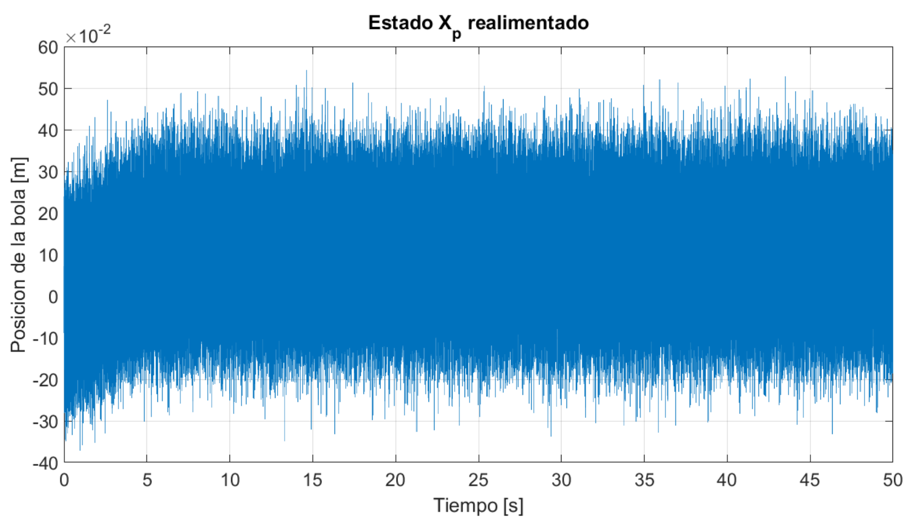
Figure 6 (b). LQG control loop, state feedback x_p with noise.



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Then, with the objective of making a closer approximation of the response of the LQG controller in the real Ball & Beam system, this control strategy was implemented in the nonlinear Ball & Beam system, as shown in the figure 7, obtaining as system response, the dynamics of the ball position described in the figure 8, which stabilized in a time of 29.79 sec and presented a steady state error of 0.28 %.

Figure 7. Simulink implementation of the LQG control loop for Quanser's Ball & Beam nonlinear system.

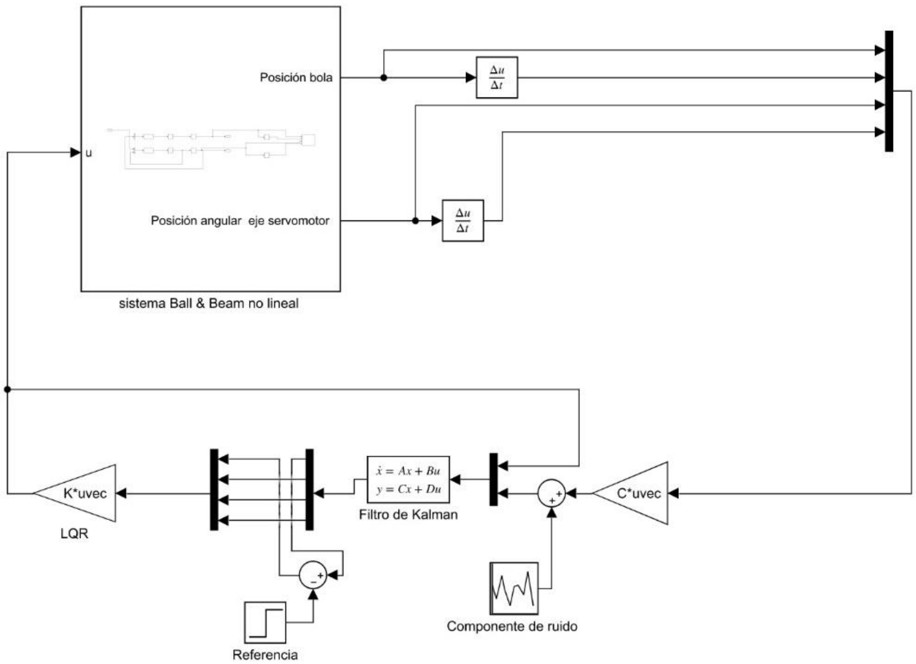

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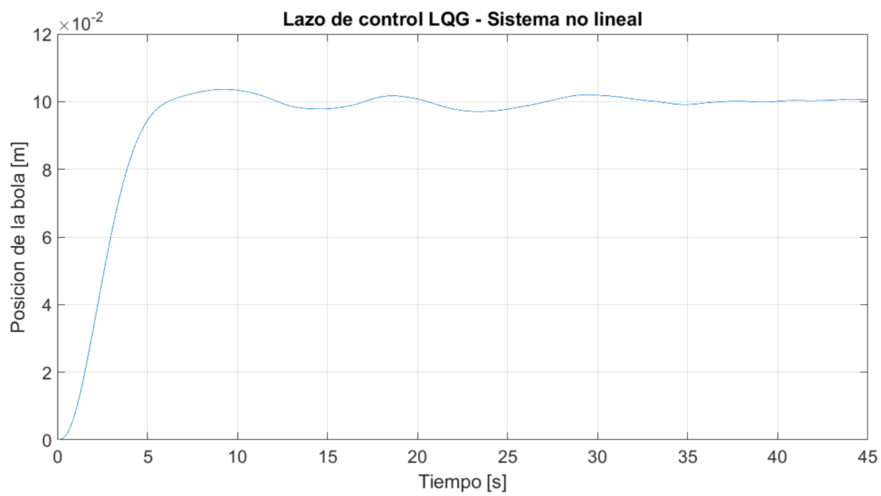
Figure 8. LQG control loop for nonlinear system, ball position control.



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As could be evidenced, the LQR and LQG control strategies manage to stabilize the Quanser Ball & Beam system, with the difference that the LQG controller minimizes the negative effects of the noise inherent to the linear position sensor of the system, and generates a much lower steady state error than the LQR controller. In order to perform a quantitative analysis of the results, the settling times and steady state errors obtained in each of the simulations performed were organized in Table 2.

Table 2. Quantitative characteristics of the responses of each control loop.

Controller	State	Settling time [sec]	Steady state error [%].
LQR	x_p	9.6446	68.38
LQG (Linear system)	x_p	12.3262	0.1523
LQG (non-linear system)	x_p	29.789	0.2836

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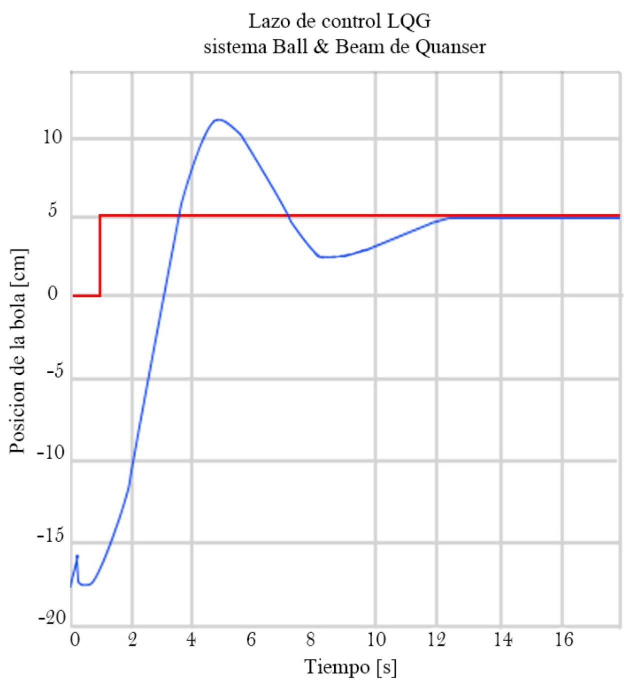
Finally, using the QUARC module of Matlab/Simulink, the LQG control loop was implemented in the Quanser Ball & Beam system, obtaining as system response the dynamics described in figure 9, where the step function is the reference imposed to the Ball & Beam system, and the other function is the dynamics of the ball position before that reference.

Figure 9. Quanser Ball & Beam LQG control loop.



Based on the response of the LQG control loop implemented in the real Quanser Ball & Beam system and analyzing the state dynamics x_p described in Figure 9, it was evidenced that the position of the ball along the beam presents a settling time around 12 seconds and a steady state error close to 0%; it is important to highlight the similarity of these data with the results obtained in the design and simulation of the LQG controller for the linear Ball & Beam system, which are present in Table 2.

This allows us to affirm that due to the correct mathematical modeling and linearization of the system around the selected operating point, it was possible to describe with great accuracy the behavior of the real system around its operating point. This, in turn, shows that the LQG control loop designed under this linear model manages to stabilize the system states both in the simulation and in the real implementation.

5. Conclusions

An LQG controller was designed, simulated and implemented for the Quanser Ball & Beam system, achieving to stabilize the position of the ball along the beam. Additionally, by means of a comparative analysis between the LQR and LQG controllers, it was validated that only the LQR controller is not efficient to control the Ball & Beam system, because the response of this control loop presents a high steady state error, even without the presence of noise in the state feedback x_p, which is associated to the measurement of the linear position of the ball. On the other hand, it was validated that the LQG controller is efficient to control the Ball & Beam system, since it manages to stabilize the system states with a steady state error close to 0%, both in the presence and in the absence of noise in the measurement of its output state; This is due to the fact that during the design of the LQG controller, both the noise associated with the process and the noise associated with the measurement of the output state are taken into consideration, thus generating an optimal estimation of all the states of the system, allowing to improve both the performance of the controller and the response of the system. For this reason, it can be stated that the LQG control technique is effective to control the Ball & Beam system, even when there is noise in the measurement of its output state.

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