UNIVERSIDAD DISTRITAL
FRANCISCO JOSÉ DE CALDAS

Visión Electrónica
Más que un estado sólido
https://doi.org/10.14483/issn.2248-4728

VISIÓN ELECTRÓNICA

A CURRENT VISION

Interpretations of the method of Descartes in didactics of algebra. Documentary study

Interpretaciones del método de Descartes en didáctica del álgebra. Estudio documental

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INFORMACIÓN DEL ARTÍCULO

Historia del artículo:
Enviado: 03/11/2021
Recibido: 06/11/2021
Aceptado: 18/12/2021

Keywords:
Descartes method
Didactics of algebra
Geometry
History of mathematics
Teaching of algebra

Palabras clave:
Método de Descartes
Didáctica del álgebra
Geometría
Historia de la matemática
Enseñanza del álgebra

ABSTRACT

This communication presents a documentary study of the of the Cartesian method in three books on the teaching and learning of algebra, its influence and use in the understanding of school algebra.

Arguments were found to indicate that some interpretations used in didactics regarding the mathematical practice of Descartes reveal a conception of the Cartesian method that privileges aspects of a syntactic and methodical type, leaving aside the analysis of the representational and semantic treatment found in the work. The educational importance given to the method of analysis is related to the algebraic manipulation and the construction of a symbolic system, the reflections on the interpretive, diagrammatic, and semantic needs of the method in solving problems are meager.

RESUMEN

Esta comunicación presenta un estudio documental de las interpretaciones del método cartesiano en tres libros sobre enseñanza y aprendizaje del álgebra, su influencia y uso en la comprensión del álgebra escolar.

Se encontraron argumentos para indicar que algunas interpretaciones usadas en didáctica respecto a la práctica matemática de Descartes develan una concepción del método cartesiano que privilegia aspectos de tipo sintáctico y metódico, dejando a un lado el análisis del tratamiento representacional y semántico que se encuentra en la obra. La importancia educativa que se le otorga al método de análisis está relacionada con la manipulación algebraica y la construcción de un sistema simbólico, las reflexiones sobre las necesidades interpretativas, diagramáticas y semánticas del método en la solución de problemas son exiguas.

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Cite this article as: J. H. Bello-Chávez, G. Mancera-Ortiz, “Interpretations of the method of descartes in didactics of algebra. Documentary study”, *Visión Electrónica*, vol. 15, no. 2, pp. 315-321, 2021. https://doi.org/10.14483/22484728.18354

1. Introduction

The participation by the History of Mathematics in Didactics implies the interpretation, selection, and analysis of aspects that the didact considers relevant for the teaching-learning processes. The didact interprets and analyzes mathematics from historical reconstructions focused on different aspects; social, epistemological, or sociological. In this way, organizes a new historical account that he considers relevant to his practice regarding a mathematical object of study. Consequently, reconstructs history from specific needs regarding the type of knowledge that wishes to work with, recognizes some historical events that inspire his action; This process is carried out from an epistemological view of the object in Didactics. This document reveals in three books on teaching and learning algebra the interpretations in relation to historical works, especially Geometry [1]. The theoretical position that the didacts of these events have made is studied from the knowledge that historians have of the work of Descartes. The interpretations and uses of the historical work are studied in a moment of the Didactics of Mathematics.

The texts were chosen because in the national context, especially in the Teacher Training Program of Mathematics at the Francisco José de Caldas, they are used to make known the historical part of the work that has been carried out in the field. The texts are in a stage of consolidation of research in teaching and learning algebra, from 1996 to 2004. In the first, Approaches to Algebra. Perspectives for Research and Teaching, part I is studied: Historical Perspectives in the Development of Algebra [2]. The second text delves into chapter eight Working Group on Algebra History in Mathematics Education of the 12th ICMI study [3]. The purpose of this chapter is to analyze studies in the history of algebra that may in the future be examined for teaching and learning. The third text Perspectives on School Algebra [4].

1.1. *What does the History of Mathematics say about Descartes' mathematical work?*

Liu's analysis reports that researchers have made two types of readings of the role of geometry and algebra in the work of Descartes [5], traditional reading and progressive reading. The first is characterized by giving more priority to Euclidean inheritance, especially the role of geometric construction. In this perspective, a greater epistemological importance to geometry is understood than to algebra. This is understood as a tool that enables the solution of geometric problems.

The progressive reading recognizes in the work of Descartes greater contribution to the development of a conception of mathematics closer to the current one. In this way, this type of interpretation recognizes less value to geometric work - without ignoring it - and emphasizes the contribution in the structuring of algebra; to the analysis and organization of abstract entities that later contributed to mathematics. In this type of reading, it is understood that Descartes released the magnitude and number of spatial intuitions.

Recognizing the differences between the approaches that have studied the mathematical work of Descartes, this study focuses the referential framework with which the documentary corpus studies from the practice that relates geometry and algebra through the method of solving problems, an idea that both recognize historical approaches to the text. The method is the center of mathematical practice that was developed through the publication of Geometry, which, as is known, is part of the Discourse on the method to conduct one's own reason well and seek the truth in the sciences [1].

1.2. *Problem solving method*

The opposition that Descartes had to the syllogistics of reasoning carried out by the scholastics is well known, especially his criticism of the convincing role to prove things that are known [1]. He sustained the need to advance in a project to find a method for discovering truths, with the idea of decomposing each geometric problem into the simplest elements [6]. Within his philosophy, this technique was aimed at guiding the solution of any geometric problem, it was based on the method of analysis and proposed a separation from the Euclidean argumentative technique:

> If, then, we wish to solve any problem, we first suppose the solution already effected, and give names to all the lines that seem needful for its construction, to those that are unknown as well as to those that are known. Then, making no distinction between know and unknown lines, we must unravel the difficulty in any way that shows most naturally the relations between these lines, until we find it possible to express a

> single quantity in two ways. This will constitute an equation, since the terms of one of these two expressions are together equal to the terms of the other. [7, p.8]

In the method the equation became a means that allowed another form of argumentation regarding the Greek tradition. Descartes' work developed the method of analysis which allowed to overcome the use of arbitrary figures that generalized the properties of objects and contributed with diagrams that show the relationships between geometric objects and allowed to specify from equations what is considered given,

> From now on, all the elements that were involved could be included in the reasoning, as if they were all given, without this unification of treatments and procedures implying at any time the least risk of confusion regarding the exact status of each one of them Gardies [8, p.3] .

Descartes raises the use of symbols early in his Geometry, noting the use of the letter to designate the magnitude from a line segment:

> Often it is not necessary thus to draw the lines on paper, but it is sufficient to designate each by a single letter. Thus, to add the lines BD and GH, I call one a and the other b, and write $a + b$. Then $a - b$ will indicate that b is subtracted from a; that is multiplied by b...[7, p.5]

This aspect helps to overcome the Euclidean vision of homogeneity. In Descartes a^2, a^3, $\sqrt[n]{a}$ $n\geq 0$, can be represented by a straight line, geometric constructions are obtained by means of segments that represent, for example, equations of degree n.

This new way of operating involved a metaphysics that in its time revolutionized the way of doing mathematics, its fundamental basis was to put aside the perspective of objects and focus on relationships [9]. Study interest in mathematics shifted from geometric objects like lines, circles, and polygons to relationships between line segments, which are understood as representatives of arbitrary quantities and can be represented by means of proportions: mathematics seen from the simplicity of the relationships between lines.

Two moments in the interpretation of the sign were established in the problem-solving method; the first one related to the interpretation of the situation, for example, in Pappus's problem part of the inferences that are made are the product of the diagram that he uses, which does not change throughout the work, the lines are in a given position, this allows us to deduce the relationships between the lines for the solution and their implications in the shape of the curves and the symbolic representations [10]. The other moment is established by geometrically constructing the line segments with lengths equal to the roots of an equation, geometrically representing the solution of equations as part of the proposed method.

According to Mancosu in the commentators of the work of Descartes two interpretations of the role of algebra in Geometry [11]. The first represented by the works of Bos, Boyer, Grosholz, Lachterman and Lenoir, which maintains that algebra is a tool that helps to provide the economy, in this way the epistemological status of the symbolic representation was that tool; the means of representation and definition was the curve. A second interpretation is that of Giusti, who considers that the identification of the curve by means of the equation is at the heart of the Cartesian program, the means of constructing curves are secondary.

Part of the work presented in Geometry is based on showing the limitations of the ruler and the compass, accepting articulated instruments to draw curves involved the means of construction in a new organization, some called by the ancients as mechanical now meet the criteria for be geometric

> If we say that are called mechanical because some sort of instrument has to be uses to describe them, them we must, to be consistent reject circles and straight lines, these cannot be described on paper without the use of compasses and ruler, which may also be termed instrument. It is not because the other instruments, being more complicated that the ruler and compasses, are therefore less accurate, for if this were so they would have to be excluded form mechanics, in which accuracy of construction is even more important that in geometry [7, p.40]

Descartes includes the treatment of mechanical instruments, compasses, which allow him to work around primary ideas about mathematics, such as the proportional mean. It is true that the geometry review leaves doubt about the existence of this type of instrument, since these could be diagrams to understand the problems; However, some commentators who have investigated Descartes' correspondence, especially with Beeckman and the treatise on private reflections, assure the existence of these instruments [12,13]. Curves were constructed that showed the relationships that were synthesized in the writing of an equation. This is a procedure that appears frequently in Geometry.

Descartes was developing a unified approach to algebraic techniques, where the problems that are susceptible to construction could be reduced to a group of standard problems, which could be represented by the relations of a known construction. He related the instruments that make it possible to draw a curve, the geometric construction and the solution of the equation, with standardized and general algebraic expressions. He showed the potential of the method to integrate problems that were separated in the classification of the ancients.

2. Analysis of the texts

The study establishes relationships between the semantic and pragmatic levels of fragments of the analyzed texts. The basic units are phrases taken between separate points or consecutive points, and textual quotations related to Geometry or regarding Descartes. A first review of the documentary corpus revealed two categories of analysis, one at the explanatory level of the historical work, the interp retations of the method, and the other at the level of praxis within the didactics, the didactic uses of the method.

2.1. Interpretations of the method

In this category of analysis, two elements of discussion stand out: the role of the symbol in method development and the role of analysis in solving geometric problems [14].

In the three texts there is evidence of a recognition and emphasis of the need for symbolic writing for the development of the method. However, there is no evidence of reference or use of geometric construction processes that guaranteed the possibility of interpreting the unknown as given. Part of the method consists of interpreting what is sought with the same characteristics and properties of the known.

The contrast between a geometric relationship and its symbolic representation was mediated by the role of construction. This aspect is recognized as the methodological purpose of Geometry, which consists in facing a crucial question in the tradition of geometric problem solving: how to build when ruler and compass are insufficient? [13].

The sections of the analyzed texts refer to an analysis devoid of geometric relationships, contradicting the perspective presented by Descartes. In later works such as Arnauld and Prestet [15], a version of the method that is closer to that presented by the didactic texts is evidenced. In phrases like the following these conceptions are evidenced:

> They observe that the history of symbolism in algebra is the invention of a system that makes it possible to solve problems by manipulation of symbols according to rules ,and without recourse to what the symbols mean. This is the legacy of Descartes and others. [3, p.10]

This conception corresponds to the progressive reading of the method. Fundamentally, it is linked to the creation of the system of mathematical signs that made it possible to structure polynomial expressions and solve problems by identifying canonical expressions:

> Thus the method continues by transforming the written algebraic expressions and the resulting equations in order to reduce them to a canonical form. This implies that it has previously been determined which expressions and which equations will be considered canonical, and that one has a catalog of all the possible canonical forms and procedures for solving each of them. [3, p.194]

In the work the canonical form of the polynomials was worked on, however, this fact is a consequence of the work on geometric aspects of the curve. The units of analysis do not allow to determine the suggested extent of the relationships between the curve and its canonical form. The figure, the construction, the development of the graph, is absent in the analyzes carried out in the text on the method of solving geometric problems. The geometric problem / construction (curve) and construction (curve) / equation pairs are fundamental in understanding the historical importance of Geometry.

Only one of the texts refers to the representation of the magnitudes and the relational work involved in the development of the method, but they do not describe the type of relationships that allow the analysis:

> In geometry, analysis revolves around the search for what is known among what seems to be unknown. The core of analysis is the hypothesis, that is the assumption that the problem is solved. As said before it imposes the development of a certain way of representing the unknown magnitudes that are considered given by hypothesis. In that process, all lines or parts of a figure are dealt with in the same way. Relations between those lines are studied, whether the lines are given or not. [4, p.36]

The analytical process is outlined by a good choice of the known and unknown in the geometric problem. This aspect is supported by the possibility that the discovery method that enables the use of algebraic techniques is covered by the understanding of the issues that precede the assumption of solving the problem [2, p.40]. The equation contains known and unknown elements; but its conformation is sustainable due to its geometric knowledge.

2.2. *Didactic use of the method*

The sections analyzed refer to the importance of the method in the acquisition of the symbolic system of school algebra. They give it relevance in the construction of a syntax for mathematics, categorizing the Cartesian method as algebraic. In problem solving the method appears related to the recognition of given and unknown quantities

> What lies at the heart of algebraic problem solving is the expression of problems in the language of algebra by means of equations. In order to be able to compare the ways of writing equations that represent word problems in different historical texts so that the comparison brings out what is pertinent for teaching, a good strategy is to take as a reference what is done in the Cartesian Method, which is the algebraic method par excellence and may be considered as the canon of the methods traditionally taught in school systems. [3, p.191]

In the didactic treatment there are no work proposals regarding the representation of magnitudes by means of segments. The difficulty that is stated in the teaching refers to the role of the symbol that underlies the use of the method. In the phases that are recognized, the interpretation of problems is not involved through constructions other than the symbolic one, ignoring the interpretive phase of the problem from the diagram that allows inferring the relationships that are represented in equations. However, different interpretations of the letter are recognized in the process of construction of the method, the passage from the unknown to the variable

> ...competent use of the Cartesian Method is linked with the creation of families of problems that are represented in the mathematical sign system (MSS) of algebra as canonical forms. This implies an evolution of the use of symbolization in which, finally, the competent user can give meaning to a symbolic representation of the problem that arises from the particular concrete examples given in teaching. Student will make sense of the Cartesian Method when they become finally aware that by applying it they can solve families of problems, defined by the same scheme of solution. [3, p.191]

The documents do not delve into the means that make it possible to understand the problems or to carry out the first step of the method, to establish the relationships of the known and the unknown. The emphasis is on the recognition of algebraic techniques, of equations as a new way of solving problems outside of arithmetic and geometric procedures. The only experiences that are presented involve problems in the arithmetic - algebra relationship and link different resources such as spreadsheets, exploration by arithmetic methods and graphs of curves, always in the construction of families of problems, in search of the understanding of the operability of the symbolic:

> The role played by intermediate methods in the passage from the classical arithmetic method to the Cartesian one has to do with the possibility of the user constructing meanings for the algebraic relationships between the elements of the problem. Although it should be pointed out that, on the other hand, the essential difference between the introduction of algebra and all previous approaches lies in that in the latter, when solving problems, the unknown is represented, although it is not operated. Inferences are made with a reference to the representation of the unknown; but if operated, this is always done by means of the data: if a mention is made of unknowns, this is only in terms of the results of operations which are being done with the data. [4, p.175]

In this category it is evident that until the date of production of the analyzed documents, the Cartesian method continued to be an element of analysis for the didactics of mathematics. On the one hand linked to the construction of symbolic systems and on the other to solving problems through the recognition of the canonical symbolic structure of polynomials [15].

3. Conclusions

Historical studies on the mathematical work of Descartes, especially Geometry [1], show a method linked to solving geometric problems and analysis. The representation of known and unknown data by means of segments, the representation of geometric relationships by equations and the study of their solution by instruments or geometric constructions, are part of the activities that characterize the practice. Construction is understood as a fundamental element in the work. The study of the properties of curves and a classification that includes new problems such as geometric, allow us to understand and use the method proposed by Descartes.

In the algebra teaching and learning texts analyzed, an idea of the Cartesian method is developed linked to the mastery of algebraic techniques and to solving problems using canonical expressions. Although the texts recognize the didactic importance of the method, the conception that persists is more linked to authors after Descartes, the algebraic technique is strengthened, and the symbolic representation of the curves does not depend on geometric approaches. No means of representation other than the symbolic are linked and the notion of curve is little worked as a means that allows us to understand the algebraic expression, in the opportunities that it appears it does so more as a representation of the symbols and not as a justification or representation of them. The interpretation made by the analyzed books rules out the use of didactic reflections on the semantics that are developed in the work of Descartes, the diagrams of the problems that allow the construction of geometric relationships and the evolution in the method of treating the letter is not They are developed within the texts. The relational spirit of mathematics in Descartes is not part of arguments that bring the historical analysis of the work closer to didactics. Correspondingly, the possibility of use to which they refer is linked to the development of the symbolic system and through it to the identification of canonical forms of problem solving; for example, in learning calculus [16-18].

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