UNIVERSIDAD DISTRITAL
FRANCISCO JOSÉ DE CALDAS

Visión Electrónica
Más que un estado sólido
https://doi.org/10.14483/issn.2248-4728

VISIÓN ELECTRONICA
A context vision

Historical considerations for the teaching of the derivative

Consideraciones históricas para la enseñanza de la derivada

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INFORMACIÓN DEL ARTICULO

Envíado: 16/11/2019
Recepción: 29/11/2019
Aceptación: 21/12/ 2019

Keywords:
Derivative Teaching,
Onto-semiotic Approach,
Representations

Palabras clave:
Enseñanza de la Derivada,
Enfoque Ontosemiótico,
Representaciones.

ABSTRACT:

Some methods that were developed throughout history to find derivatives from Apolonio de Perga to Cauchy are presented, then itineraries are proposed for the introduction of the derivative in the curriculum: addressing positions of the theory of Action, Process, Object and Scheme (APOE) and the tools proposed by the known as the Onto-Semiotic Approach (OSA), for elements of the derivative before the limit and limits before derivatives. From these, semiotic representations are proposed that allow to approach the derivative (numerical, algebraic, formal, geometric, infinitesimal, local, variational and computational related approximation). Finding that the concept as taught today does not coincide with the development of its genesis and that a way of understanding the concept of derivatives from the use of treatments and conversions of registers of representation.

RESUMEN

Se presentan algunos métodos que se desarrollaron a través de la historia para encontrar derivadas desde Apolonio de Perga hasta Cauchy. A partir de estos, se plantean representaciones semióticas que permiten abordar la derivada (numérico, algebraico, formal, geométrico, infinitesimal, aproximación afín local, variacional y computacional). Encontrando que el concepto como se enseña en la actualidad no coincide con el desarrollo de su génesis y que una forma de comprender el concepto de derivadas a partir de uso de tratamientos y conversiones de registros de representación.

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Cite this article as: W. Gordillo-Thiriat, W. J. Pinzón-Casallas and H. Vacca-González, "Historical considerations for the teaching of the derivative", Visión electrónica, algo más que un estado sólido, vol. 2, no. 2, Special edition, july-december 2019. DOI revista: https://doi.org/10.14483/issn.2248-4728

1. Introduction

The historical studies of mathematical objects [1] provide elements that refer to the different conceptions of mathematical objects, in which the possible difficulties that arose in their construction are analyzed. The derivative, for example, has been motivated in history by two types of fundamental problems, the first of a geometric nature, particularly related to tangents and normal (amounts of change), as well as to maximum and minimum values, the second problem on its part deals with everything related to velocity, particularly instantaneous velocity (increments).

A historical reconstruction of the antiderivative object [2], replicated for the derived object I have revealed that as regards the tangent lines at a particular point of a curve, it can be pointed out that the first known attempts are Apolonio de Perga (262 - 190 B.C.), who gives rise to ten possible cases, from the simplest two in which the three elements are three points or three straight lines, to the most difficult of all, to draw a tangent circle to three other given [3].

Sometime later, René Descartes (1596 - 1650) points out that the great importance of the relationships between the properties of a curve and its equation, evidencing the interest and importance of dealing with problems such as the determination of the normal to an algebraic curve at any point, and many other similar problems known today have a strong relationship with the derivative and phenomena of variation and movement. After this Pierre de Fermat (1601-1665) published in a memory in 1635 and entitled Methodus and Disquirendam Maximan et Miniman de Fermat, as well as the relationship of the same with phenomena related to movement and variation.

Roberval (problems related to instantaneous velocity), Torricelli (problems related to maximums and minimums, as well as to the tangent that he published in 1644 in a book titled De Parabole, to which he added as an appendix both the quadrature of the cycloid and the construction of the tangent [3]) and Isaac Barrow (tangent method), each of whom contributed his work significantly to the process, Newton (1642-1727) recognized that Barrow's algorithm was no more than Fermat's slightly improved [3].

Within some of the contributions that Newton worked on, he presents a wide explanation of the relationship between the curvature of a curve and its ordinate, that is, he manages to establish that if the curvature of a curve is given by (1),

$$a\ x^{m} \tag{1}$$

where in (1), m is an integer or fractional value. The rate of change of the area with respect to x is given by (2)

$$max^{(m-1)} \tag{2}$$

in other words, sets out the elements you will later use in your flux and fluent calculations.

For his part, Leibniz (1646-1716) independently of Newton, also worries about some matters related to infinitesimal calculus, in a series of particular notes dated November 1675, establishes that a curve can be expressed as a discrete succession of values of the ordinate and to which it is possible to correspond a discrete succession of the abscissa x and the ordinate ones y; but perhaps one of the most important contributions of Leibniz and his followers (Bernoulli brothers, Marquis of L'Hopital, Euler,...) consisted in the notation and language of differential calculus, in which they called differential of a magnitude *(dy)* the infinitesimal variation of that magnitude *(y)* (its "moment", in Newton's words). If *dy* had been able to take a macroscopic value, it would not have coincided with Δy but, as it was only given infinitely small values, in that range it identified itself with Δy without making any mistake. Thus, the differential of the position *(de)* although in macroscopic terms it did not correspond to any displacement, could be identified with the displacement occurred in an infinitely small-time interval *(dt))* [4].

They defined the derivative as the quotient of very small increments and to calculate it they considered that an infinitesimal variation *dx* would also produce an infinitesimal variation *dy* then they divided both members by *dx* and only at that moment they despised the infinitesimal summands, thus obtaining the derivative

To calculate the derivative of function (3), it is taken into account that an infinitesimal variation dx would also produce an infinitesimal variation *dy*,ie (4).

$$y = x^2 \tag{3}$$

$$y + dy = (x + dx)^2 \tag{4}$$

When dividing by dx, we have (5), and since here we despise infinitesimal summands, we obtain (6)

$$\frac{dy}{dx} = 2x + dx \tag{5}$$

$$\frac{dy}{dx} = 2x \tag{6}$$

The contributions of Newton and Leibniz connected the problems of mechanics with those of geometry, thanks to the method of coordinates, which offers a graphical representation of the dependence between two variables; that is, functions can be represented graphically; they also demonstrated the rules for graphically; they also demonstrated the rules for deriving the product and the quotient of functions as they are known today.

Lagrange published in 1784 a theory of the functions of the theory of the analytical functions liberating the differential calculation of the infinitely small and changed the notation to refer to the derivative, replacing the expression (7) with f'

$$\frac{df}{dx} \tag{7}$$

Lagrange defined the derivative as the linear coefficient of the series development of powers of a function around a given point, i.e. (8).

$$f(x) = f(a) + f'(a)(x-a) + f''(a)(x-a)^2(2!) + \cdots \tag{8}$$

This definition uses the idea of tangent as a particular form of contact between a line and a curve. To calculate the derivative developed at f around the x point with an increment h, you get (9) equaling you get (10), which is equal to (11)

$$f(x+h) = \sum_{i=0}^{n} C_i^n i!\, x^{n-i} h^i \tag{9}$$

$$\sum_{i=0}^{n} C_i^n i!\, x^{n-i} h^i = C_0^n 0!\, x^n + C_1^n 1!\, x^{n-1} h + \cdots + C_{n-1}^n (n-1)!\, x h^{n-1} + C_n^n n!\, h^n \tag{10}$$

$$C_0^n 0!\, x^n + C_1^n 1!\, x^{n-1} h + \cdots + C_{n-1}^n (n-1)!\, x h^{n-1} + C_n^n n!\, h^n$$
$$= a_0(x)0! + a_1(x)h(1!) + a_2(x)h^2(2!) + \cdots a_n(x)h^n(n!) \tag{11}$$

If you write the coefficients in terms of those derived from f , you have (12)

$$f(x+h) = \sum_{i=0}^{n} C_i^n i!\, x^{n-i} h^i = f(x) + f'(x)h + f''(x)h^2 + \cdots + f^{(n)}(x)h^n \tag{12}$$

The following is an example that illustrates this strategy

- To calculate the derivative of function (13)

$$f(x) = x^3 \tag{13}$$

take into account that f develops around point x as an increment h, i.e. (14)

$$f(x+h) = \sum_{i=0}^{3} C_i^3 i!\, x^{3-i} h^i \tag{14}$$

By developing it results (15)

$$C_0^3 0!\, x^3 + C_1^3 1!\, x^2 h + C_1^3 2!\, x h^2 + C_1^3 3!\, h^3 \tag{15}$$

And therefore, you get (16)

so, the derivatives of f are the quotients of h in the previous expansion, i.e. as shown in (16)

$$x^3 + 3x^2 h + 3x(2!)h^2 + (3!)h^3 \tag{16}$$

which is equivalent to (17)

$$f'(x) = 3x^2, f''(x) = 6x\,, f'''(x) = 6 \tag{17}$$

Finally, among the last steps of differential calculus is that of the rigor that is acquired thanks to the competition of mathematicians such as Cauchy who imprints a certain rigor on it. in the first half of the 19th century, who formulated a precise definition of the infinitesimal quantities of the derivative and the integral. As for the differential, it ceased to be identified as an infinitesimal increment, was emptied of all physical meaning and went on to occupy a marginal place in the structure of the calculation. [4]

Cauchy defined the derivative as the limit of the incremental quotient, i.e. (18)

$$\frac{df(x)}{dx} = \lim_{n \to a} \frac{f(x) - f(a)}{x - a} \tag{18}$$

This definition puts the tangent as the result of a particular process at the limit, then the explanation of the nature of the tangent rests more on the process of obtaining it than on the tangent property itself. That is, the tangent is the result of the process.

2. Conclusions

The itineraries presented below have been treated and analyzed by Badillo [5], in which he contemplates the treatment of the semiotic complexity associated with the notations used in the definitions of macro objects $f'(a)$ and $f'(x)$

he first itinerary follows the historical genesis of the concept (the one derived before the limit) which has two ways of presenting it:

The macro object $f'(a)$ is defined first and then the macro object $f'(x)$

First define the macro object $f'(x)$ and then the macro object $f'(a)$

The second itinerary is the concept of limit before derivative, which can be presented in two forms:

The macro object $f'(x)$ is defined first and then the macro object $f'(a)$ First define the macro object $f'(a)$ and then the macro object $f'(x)$

On the other hand, mediation using technology - virtual training platform- will be a perspective that complements the historical sense presented here, about the teaching of the derivative, and that will be the object of investigation forward, [6-10].

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