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Characterization model of asphalt mixtures using digital image

processing

Modelo de caracterización de mezclas asfálticas usando herramientas

digitales

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Abstract

The construction of roads and highways are the central axis for the development of cultural, economic, and social growth of a country. There are different types of structural materials used for the construction of these roads, among which the use of asphalt mixtures for the creation of flexible pavements stands out. Este work shows the development of to characterization model for asphalt mixtures through the implementation of different digital tools to know, through an image, a detailed description of the asphalt mixture, allowing to simplifying decision making in the field, optimizing the process of construction and quality of pavements. For the development, digital image processing

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algorithms, discrete Fourier transform, and Kohonen self-organizing maps are implemented. The results Show the obtaining of the different characteristics, the error less than 4% of the digital granulometry with respect to the one used in the laboratory and the relationship of the segregation with the mechanical behavior of the mixtures. in future work, the validation of the incidence of the other parameters calculated with the mechanical properties of the material es projected.

Keywords: Kohonen, pavements, characterization models, digital image processing.

Resumen

La construcción de vías y carreteras son el eje central para el desarrollo y crecimiento cultural, económico y social de un país. Existen diferentes tipos de materiales estructurales usados para la construcción de estas vías, dentro de los cuales se resalta el uso de mezclas asfálticas para la creación de pavimentos flexibles. Este trabajo muestra el desarrollo de un modelo de caracterización para mezclas asfálticas mediante la implementación de diferentes herramientas digitales con el objetivo de conocer mediante una imagen, una descripción detallada de la mezcla asfáltica, permitiendo simplificar la toma de decisiones en campo, optimizando el proceso de construcción y calidad de los pavimentos. Para el desarrollo se implementan algoritmos de procesamiento digital de imágenes, trasformada discreta de Fourier y mapas auto organizativos de Kohonen. Los resultados muestran las diferentes características obtenidas, el error menor al 4% en la construcción de la granulometría digital con respecto a la usada en el laboratorio y finalmente, la relación de la segregación con el comportamiento mecánico de las mezclas. Como investigación futura se proyecta la validación de la incidencia de los otros parámetros calculados en este trabajo, con propiedades mecánicas del material.

Palabras clave: Kohonen, pavimentos, modelo de caracterización, procesamiento digital de imágenes.

1. Introduction

Pavement structures are the fundamental axis for the development and transportation of goods, the supply of the regions and the movement of the goods depend on the roads. national economy [1]. Going deeper into the study of this type of structures allows us to improve his design, construction and maintenance, doing roads that toast top benefits to level economic, comfort and of duration [2].

Within the pavements there are flexible pavements, which are used as binder a material called asphalt. Asphalt is the bottom of the crude oil distillation process. Mixtures are created with asphalt, aggregates (rock fragments) and different additives. asphalt the which they use for the construction of the pavement in mention [3][4].

For the characterization mechanics of the mixes asphalt they use a series of essays of laboratory for know his behavior and also his structure internal. Many of these Tests are high-cost processes and require a long time to execute and analysis [5]. In the last years for the study of these materials they have generated different digital tools that, through analytics, image processing and intelligence artificial, they have accomplished parameterize the behavior of the mixes to leave of all the data and images obtained and his prosecution digital [6].

Different jobs they have developed Models for extract characteristics of the aggregate and his distribution inside of a mix asphalt through images and they have arrived to correlate some of these characteristics with the mechanical behavior of the material. Inside of these jobs finds the developed by Dengao et *to the.* [7] in where through the acquisition and prosecution of a image, apply a model of items finite for create stress distribution and determine how the material behaves in the face of changes Of temperature. The authors

obtain that indeed the homogeneous distribution of the aggregates It allows that the sample medium greater loads. By other part, liwan Shi et *to the*. [8] develop a process for the segmentation of the aggregate of an asphalt mixture using images and analyze the contacts between aggregates, it former with the aim of correlate these contact values with the resilient modulus laboratory test, finding that Particles above 2.36 mm have the most impact on performance. For another side, Junyao et *al* . [9] perform the digitization of the aggregate of an asphalt mixture, and create a finite element model where they analyze the structure and shape of the aggregate with the purpose of characterizing the cracking of the mixture. In such a way that they can recreate shape virtual the trials of laboratory with mistakes by below of the 10%.

In this work, an image characterization model for mixtures is presented. asphalt, in where through a only Photography they can extract 41 parameters different. The above, with the objective of being able to create a robust model that is capable of process a large amount of information that allows micro and Structural meso of an asphalt mixture. Additionally, with some of the parameters calculated valid the granulometry, the segregation of the aggregate and his relationship with the dataexperimental obtained in the laboratory.

2. Methods and materials

The work model used to develop the study is seen in Figure 1. In where it begins with the manufacturing of the samples, followed by an acquisition process of images and segmentation of the fragments of rock (added) in different images by size groups (sieves). Once the aggregate is segmented, the process of characterization of the sample in two parts. The first part is the characterization of the added individually and the second taking into account the entire structure. Finally The results will be reviewed and some parameters will be validated obtained with the Data of laboratory physical.

Figure 1. Methodology of development of the job.



2.1. Design and manufacturing of the mixes asphalt.

For the design and manufacture of the different asphalt mixtures, the curves were used granulometric (graph that indicates the percentages of aggregates of each size that contains the mixture) of Figure 2 stipulated in the technical specifications of the Institute National of Tracks (INVIA) [10]. The mixes manufactured they mention to continuation:

- Mix dense in hot. Code: MDC-19 [eleven].
- Mix open in hot. Code: MDren [12].
- Mix dense in hot with Recycled material. Code: MDC-19-RAP.[13].
- Mix dense in hot with natural asphalt. Code: MDC-12-ASF [14].

Figure 2. Granulometric curves and manufacturing of asphalt mixtures.



In the end there are a total of 20 test tubes for each mixture. They are cut in half and photographs of its flat face are captured, subsequently the SCB test [10] is executed, which It allows characterize some properties mechanics of the mixtures.

2.2. Segmentation of the aggregate.

The acquisition and segmentation of the image is carried out using the protocol used in [15], using an 18 mega pixel camera located 20 centimeters from the sample and illuminated with white LED light. There are tones in the aggregates of the mixture that affect the selection of a single threshold for a binarization. For this reason, the method is used of Otsu to select the threshold [16]. After binarization, it is necessary to relate the actual size of the sample with its size in pixels of the photo. The above is done measuring three heights of the test tube physical and three heights of the digital, for subsequently calculate the value of K (average of heights real divided by the average of the heights of the photo). The parameter K is a variable that relates the dimensions physical of the test tube with his image. The process former observe in the Figure 3.





Source: own.

Through digital processing techniques, opening morphologies are applied that allow separate the material from the image. In the figure 3 observe the impact that have different values of the structure morphological in the separation of the aggregate (a disk is taken as a structure because it fits better to the contours of the aggregate in the image).



Figure 4. Morphology of opening with different values of radio of the structure of disk.

Source: own.

By using morphological operations, the elements of the image are modified and even some are eliminated, for this reason using the Pearson correlation expression [17] Verify that when comparing the resulting image and the original, the modification is not greater than the 5%. If this correlation is greater than 95%, two additional processes are carried out. The First is creating 9 images or sieves, for each particle size group using the parameter K (The size of the groups and name of the sieves used are the same as those used in the laboratory for the construction of the granulometric curve). The 9 Sieves will be the input for the characterization of granulometry and segregation. In second place, makes a individualization of the aggregates, already that these they will be the entrance for the description of the parametery, segregation and description of the aggregate are the parameters that determine the characterization model. Figure 5 presents a example of the separation by cluster and individual of a mix MDC-19.

Figure 5. Screening digital and individualization of the aggregate of mix MDC-19.



Source: own.

2.3. Model of characterization

Inside of the model of characterization they get parameters necessary for the description meso and microstructural of the mix asphalt. These parameters they have been selected according to their usefulness within the parameterization of the properties mechanics of the mixtures. These parameters are:

- Granulometry: The aggregates separated in the 9 sieves they use for the creation of the curve granulometric digital.
- Aggregate segregation: Segregation is a parameter that measures the distribution final of the particles inside of the mix asphalt.
- Description of the aggregate: The characterization of the shape and texture of each of the aggregates is carried out by measuring the primer parameters (indices of shape), second (irregularity of form) and third order (texture) [18].

The model of characterization has as entrance the aggregate segmented individually and the 9 images of the sieves. In the Figure The structure of the model is shown. characterization of the mix asphalt.



Figure 6. Model of characterization.

2.4. Description of the Aggregate

The model for the extraction of the characteristics of the aggregate presents in the Figure 18, this figure sample the aggregate individualized. So as also the processes additional for the calculation of the parameters of characterization. Inside of these processes additional they use the equations of the Table 1, the transformed discreet of Fourier (DFT) and the classification of Krumbein.





Order	Parameter	Equation		Parameters	Autho r
First	Circularity	$C = \frac{4\pi A}{p^2}$	(1)	(TO) area, (P) perimeter of the aggregate.	Riley
Second	Angularity	$A = (180 - a) \frac{x}{R_{ax-in}}$	(2)	R_{ax-in} radio of the circle signed up, x distance of the center	Lees
				of the circle to the spot further far a angle measured	
Second	Symmetry	$R = \frac{(D_{s1} + D_{s2})/2}{D_{l}}$	(3)	D_{I} diameter circle signed up, D_{s1} and D_{s2} diameter of the	Swan
				two corners further acute	
Third	Roughness	$Ru = \frac{P}{-\pi D_{avg}}$	(4)	<i>P</i> perimeter, D_{avg} diameter average.	Janoo
First	Circularity	$C = \frac{P^2}{4\pi A}$	(5)	TO area, Q perimeter of the aggregate.	Janoo
First	Rectangularity	$Re = \frac{A}{A_r}$	(6)	A area of particle, A_r area of rectangle framed	Kwan
Second	Index of shape	$RA = \frac{\underline{M}_{a}}{\underline{m}_{a}}$	(7)	M_a axis elderly, m_a axis minor of the particle.	Masad
Second	Roundness	$R = \sum_{n=1}^{n} R_{n} \theta_{n+1} + S_{n} R_{n}$	(8)	$R_{\theta+5}$ and R_{θ} radio of the particle in address θ with and	Masad
		$\overline{\theta}=0$ K_{θ}		without increase.	

Table 1. Equations of description [19][20][21].

Source: own.

By applying the equations of Table 1 on the individualized aggregates, numerical characterization. First of all, the measurement of diameters, axes and polygons, for apply subsequently the equations. The orientation of the aggregate with respect to major axis decade aggregate varies between 0° to 90°. The procedure is applies to each one of the aggregates separated. In the Figure presents a example of the measurement.



Source: own.

For the implementation of the transformed discreet of Fourier, extract fragments of the grayscale image of the individualized aggregates, to center in the image only the surface texture. Then, applying the Fourier transform, we obtain the intensity spectrum that parameterizes the texture of the image. In the difference of the transform for two is observed types of aggregates where one presents a more uniform intensity compared to the other. To obtain the texture index, the magnitude of the complex coordinates is calculated and real intensity image. The increase in the texture index refers to surfaces with elderly irregularity.



Figure 9. Extraction of the image of work and application of the transformed.

Source: own.

To generate the Krumbein roundness index, an auto map is implemented. Kohonen organizational structure (SOM) with 100 neurons [22-29]. Which learns the 81 aggregates designed by Krumbein to then be able to classify the new ones. For the classification of index of each one of the aggregates couple a grid neural MLP that has as function interpret the map of activation. The map of activation of the SOM and a example of classification presents in the Figure 10.



Figure 10. Map of activation aggregate mix MDren.

Source: own.

2.5. Reconstruction granulometric

Taking as input the 9 separate images with the particles of different sizes obtained within the segmentation process, an area value is calculated per image product of the sum of the area of each of the particles found in that image. Then, calculate the percentage sieve in function of the addition total of the areas of all the particles of the 9 images. This worth by image compare with the percentages real (In the laboratory, particles are separated into the same size groups to be able to carry out the design of the mix), this process known as the design of the curve granulometric. The previous procedure is carried out with the 20 halves of samples for the four mixtures used (MDC-19, MDC-19-RAP, MDren and MDC-12-ASF). The reconstruction of the different grain size curves is presented.



Figure 11. Reconstruction granulometric.



2.6. Segregation of the Mix

An ideally constructed asphalt mixture should have a distribution where the Vector sum of the areas of the aggregates with respect to the center of the mixture be equal to or close to 0. Otherwise the mixture presents a segregation of its aggregate (Sg_{Index}) with a magnitude ||XY|| in a direction θ [30]. To get the resulting vector for the aggregate of each sieve the definition of center of mass in where each particle has a weight of agreement to his location in the flat and his dimension [31]. The vectors resulting compare inside of a series of circumferences, that show the deviation from the center until the radio elderly that can be

obtained within the sample. Additionally, the final vector that calculate with the same method of center of mass wearing the vectors resulting and the area average for each sieve. It previous illustrates in the Figure .





3. Results

According to the research developed, the model of characterization for the asphalt mixtures delivers 41 parameters within which are: 12 graphs of distribution of aggregate shape and texture parameters, 9 material percentages, 9 magnitudes and 9 angles of the images of the sieves, and 2 of the segregation total consolidated.

The parameters calculated for description of the aggregate they organize in graphics independent (12, a by each index calculated). Each graph this composed by 8 ranges and 8 percentages, the ranges are obtained with the maximum and minimum value of each of the indexes and subdividing them into 8 groups of values. The percentages are calculated, just like for the segregation of the aggregate, through the number of the aggregates that belong to each range in function of the aggregate total. The consolidated of characterization of a mixMDC-19 presents in the Figure .

Source: own.



Figure 13. Characterization of the aggregate inside of the mix.

Source: own.

The second part of the characterization model is the granulometric reconstruction and the calculation of the aggregate segregation index. The first thing to do is validate the error in the reconstruction of the granulometry based on what was detected in the photo and its real laboratory values. The error in the comparison of laboratory granulometry and digital of the Figure sample that the method in general drive a mistake lower to the 4% in the reconstruction granulometric, excluding the aggregates fine happens sieve No 200. In the sieve No. 200 particles measuring 0.074 mm on each side are found, so they can be easily detected. through the process of acquisition and the camera used No is efficient. The

mistake lower to the 4% is taken as a permissible error if it is taken into account that the recovery method of the added for the reconstruction of the granulometric curve, through the extraction of asphalt with centrifuge drive a margin of the 1 to the 4% [32]



Figure 14. Mistake in the reconstruction granulometric.

On the other hand, the segregation index of the mixtures is used in this work to validate preliminarily his relationship with the behavior mechanic of the different guys of manufactured mixtures. A total of 80 SCB tests were run, 20 specimens for each mixture (MDC-12-ASF, MDC-19-RAP, MDC-19 and MDren). The results between segregation and the experimental variables measured (maximum load supported by the sample before breaking and area under the curve of the graph resulting from the test) are presented in Figure . The graphics show as the parameter of segregation No it affects significantly the burden maximum neither the area low the curve of the mix of asphaltite. This behavior can be directly related to the granulometric characteristics of the mixture as it is a material consisting mainly of very fine particles [33]. Similarly, when analyzing the behavior of the maximum load of the other mixes, it is evident that the incidence of the segregation index in the mixes is higher than average than in the mixes with coarser aggregates.

Source: own.







Thus, it is observed that this digitally calculated parameter does have an impact on the mechanical behavior of an asphalt mixture as also supported in [30], in future works it is planned to do this validation with the other calculated parameters.

4. Conclusions

The model of characterization done has accomplished a integration between techniques conventional (numerical characterization and digital image processing) and parameters calculated from artificial intelligence techniques (Krumbein roundness). It is projected use this model as the input to models of prediction with which are made preliminary estimates of behavior mechanic of a mix.

The model for the granulometric reconstruction of asphalt mixtures of different types using images proves to be efficient with an average normalized error of 1.52% up to the No. 80 sieve, taking into account that the No. 200 sieve presents an average error of 30% in segmentation. The algorithm detects coarse aggregate with 96% accuracy, medium and part of the fine, which makes it a versatile tool and easy to apply. His implementation would serve as an aid tool in the quality inspection and design of pavements, since it only requires

an image of the sample. Likewise, being able to calculate These values are essential since granulometry is a fundamental part of resistance. mechanics of a mix asphalt.

From the correlation graphs it is evident that the homogeneity in the distribution of the Adding an asphalt mixture has an impact on the strength of the sample. Can preliminarily validate one of the parameters determined in this study and some mechanical properties of the material, project future works where other laboratory tests and will use the other calculated parameters to find new correlations through the model generated.

Recognitions

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