Analysis, design and simulation of a sliding mode control for a parallel robot with 3 degrees of freedom 3SPS-1U

Análisis, diseño y simulación de un control por modos deslizantes para un robot paralelo de 3 grados de libertad 3SPS-1U

Katherin Duarte-Barón¹, Jhon Jairo Gil-Peláez², Carlos Borrás-Pinilla³

Abstract

Within the study of robotics there are parallel robots, which are revolutionizing the world of automation thanks to their advantages in precision, speed, safety and versatility. These robots are used in the automotive, aeronautical, manufacturing, medicine, and research sectors, among others. For this reason, a sliding mode control is designed and implemented for a 3SPS – 1 U parallel robot, starting with an investigation of information recorded in articles, books, monographs and other material available in databases corresponding to the topic. Subsequently, the mathematical models of the robot, the mobility analysis, the kinematical and the dynamics of the system are analyzed, which allows finding a non-linear mathematical expression on which the robust control strategy by sliding modes is designed, implementing it in Matlab. Finally, the performance of the sliding mode control strategy is compared with another robust strategy: computed torque control, the best performance is presented with sliding modes control.

Keywords: Control, Dynamics, Kinematics, Parallel robot, Sliding mode control.

¹ Engineer in Mechatronic, University Santo Tomas, Colombia, master's in mechanical engineering, University Industrial of Santander, Colombia. Professor: University Santo Tomás, Colombia. E-mail: <u>katherinduarte@ustavillavicencio.edu.co</u> ORCID: <u>https://orcid.org/0000-0002-3649-2807</u>

² Engineer in Mechanical, University of Valle, Colombia, master's in civil engineering, master's in mechanical engineering, Ph.D. in civil engineering, University of Puerto Rico - Mayagüez, Puerto Rico. Professor: University Santo Tomás, Colombia. E-mail: <u>jhongil@ustavillavicencio.edu.co</u> ORCID: <u>https://orcid.org/0000-0001-7570-2558</u>

³ Engineer in Mechanical, University Industrial of Santander,, Colombia, master's in mechanical engineering, Ph.D. in mechanical engineering, University Of Oklahoma, United states. Professor: University Industrial of Santander, Colombia. E-mail: <u>cborras@uis.edu.co</u> ORCID: <u>https://orcid.org/0000-0002-1014-2817</u>

Resumen

Dentro del estudio de la robótica se encuentran los robots paralelos, que están revolucionando el mundo de la automatización gracias a sus ventajas en precisión, velocidad, seguridad y versatilidad. Estos robots se usan en el sector automotriz, aeronáutico, manufacturero, en medicina, en investigación entre otros, por esa razón se diseña e implementa un control por modos deslizantes para un robot paralelo 3SPS – 1 U, iniciando con una investigación de información registrada en artículos, libros, monografías y demás material disponible en bases de datos correspondientes al tema. Posteriormente se analizan los modelos matemáticos del robot, el análisis de movilidad, la cinemática y la dinámica del sistema, lo que permite encontrar una expresión matemática no lineal sobre la cual se diseña la estrategia de control robusto por modos deslizantes, implementándola en Matlab. Finalmente se compara el desempeño de la estrategia de control por modos deslizantes con otra estrategia robusta: el control por par computado, siendo la de modos deslizantes la que mejor desempeño presenta.

Palabras clave: Cinemática, Control, Dinámica, modos deslizantes, robot paralelo.

1. Introduction

Robotics has increasingly entered the industry, and today it is very common to encounter robotic manipulators of any kind in different industries. For this reason, research centers and universities delve into topics related to this area, from its mathematical model, analysis of the workspace, singularities, and especially the development and implementation of control laws that fulfill a specific task [1].

The most studied robots are serial ones; however, in recent years, parallel robots are being used in the industry due to their advantages over serial manipulators, such as greater rigidity in their structure, higher precision, the ability to reach high speeds, and the capacity to support large loads [2].

These robots are used in the automotive [3], aerospace [4], manufacturing [5], medicine [6], and research sectors, among others [7], which has allowed the establishment of configurations, classifications, mathematical models to analyze mobility, kinematics, and dynamics [8], and various control actions have even been proposed for different types of robots [9].

In this document, the design and simulation of a robust sliding mode control are shown, initially analyzing a classic system, mass-spring-damper, and subsequently implementing it in a 3SPS-1U parallel robot, considering that its dynamics have already been previously analyzed [9]. Simulations are carried out in MATLAB with different trajectories and comparing the performance with a robust strategy widely used in robotics, computed torque control, to determine which one presents a better response.

2. Methodology

To achieve the proposed goal, the methodology shown in Figure 1 is established





Source: Own.

In the first stage, information is gathered from articles, books, monographs, etc., related to the topic to establish the various developments throughout history. The different control strategies

implemented in various types of robots are analyzed. Subsequently, a sliding mode controller is designed for a mass-spring-damper system, verifying its performance to apply it to the dynamic model of the parallel robot under study. Then, it is drawn in SolidWorks and later exported to Simulink to preserve its physical and geometric properties, allowing the controller's performance to be observed in a realistic manner. Once the controller is designed, simulations are conducted, and the results are analyzed by comparing them with another control strategy. Finally, conclusions are made based on the obtained results.

3. Previous research

Parallel robots represent a special class of robots that have sparked significant interest in the scientific and engineering communities throughout history. These systems, characterized by their configuration of multiple kinematic chains connected to a mobile platform, offer notable advantages in terms of precision, speed, and load capacity. Over the years, various control strategies have been developed to optimize their performance in a wide range of industrial. medical, and space applications [10]. In various types of robots, control strategies such as Computed Torque Control [11], Sliding Modes [12], Adaptative Control [13], Fuzzy Control [14] and predictive Control [15], have been implemented, among others, showing good performance. This is because these techniques work with the model, do not require linearization, and respond well to uncertainties. However, the classical PID control technique has not been neglected, particularly at the industrial level, providing an acceptable response for specific requirements [16]. The research carried out has allowed us to establish that some of the mentioned control strategies have been implemented for parallel robots, however, for the configuration under study, there is no documentation that shows the application of strategies such as sliding modes, adaptive or Fuzzy control.

For the model under study, 3SPS – 1U (three degrees of freedom, spherical joint, prismatic, spherical, with central mast universal joint), mobility, kinematics [17], dynamics have been

analyzed and the strategy of computed torque control [9] so that the complete development of the dynamic equations necessary to implement the sliding mode control strategy is already documented.

Sliding Mode Control (SMC) is a robust control technique. In this strategy, a control law is defined that switches at high frequency to bring the system's state to a hyperplane, known as the sliding surface, with the objective of maintaining it there despite possible disturbances with the objective of maintaining it there despite possible disturbances [18]. This type of control has been used in serial robots and, in recent years, has been implemented in parallel robots. One of the significant advantages of designing the controller in this manner is that the effect of the nonlinear terms present in the plant's dynamics is considered as disturbances - uncertainties and is completely rejected. Additionally, with this type of control, the system is forced to behave like a first-order system, which ensures that no overshoot will occur when regulating the system from an arbitrary initial displacement to the equilibrium point [19]. In the design of a Sliding Mode Control, the dynamic of the sliding surface is established, subsequently, the stability and the existence of the sliding mode are evaluated through a control law that ensures a sliding regime [20]. One of the problems when working with this control strategy is the chattering caused by the sgn function. However, the solution to this problem involves using the sat function.

4. Controller Design

For this case, it is necessary for the mobile platform to be positioned in a specific manner, so each actuator must adopt the ideal configuration. This ensures that the end-effector (mobile platform) achieves the required position and additionally rejects disturbances.

Given the similarity between the dynamic equation of the robot and that of a classical massspring-damper system, the development of the control law for this system is first presented, and then extrapolated to the robot. The characteristic expression of a mass-spring-damper system is given by equation (1), where m is the mass, b is the damping coefficient, and k is the stiffness coefficient.

$$m\ddot{x} + b\dot{x} + kx = f \tag{1}$$

which for a coupled system of n degrees of freedom, can be represented through matrix as shown in equation (2)

$$[M][\ddot{q}] + [B][\dot{q}] + [K][q] = [F]$$
(2)

Now, considering that the sliding surface equation is defined as (3),

$$S = \left(\frac{d}{dt} + \lambda\right)^{n-1} \tilde{q} \tag{3}$$

where λ is a positive constant that geometrically represents the slope of the sliding, *n* is the order of the system and \tilde{q} corresponds to the tracking error, defined as $\tilde{q} = q - q_d$, since the system under study is of second order, the sliding surface is given by (4),

$$S = \tilde{q} + \lambda \tilde{q}$$

$$S = \dot{q} - \dot{q}_d + \lambda \tilde{q}$$
(4)

To ensure that the system state remains on the sliding surface, it is necessary to select a control law such that $\dot{S} = 0$, so differentiating the expression (4),

$$\dot{S} = \ddot{q} - \ddot{q}_d + \lambda \ddot{\tilde{q}} \tag{5}$$

Now, solving the highest order derivative from equation (2),

$$[\ddot{q}] = [M]^{-1}[F] - [M]^{-1}[B][\dot{q}] - [M]^{-1}[K][q]$$
(6)

And expression (6) is replaced in (5),

$$\dot{S} = [M]^{-1}[F] - [M]^{-1}[B][\dot{q}] - [M]^{-1}[K][q] - [\ddot{q}_d] + \lambda [\dot{\ddot{q}}]$$
(7)

Like $\dot{S} = 0$

$$0 = [M]^{-1}[F] - [M]^{-1}[B][\dot{q}] - [M]^{-1}[K][q] - [\ddot{q}_d] + \lambda \left[\dot{\tilde{q}} \right]$$
(8)

In equation (8) la F represents the control signal, taking this into account, said variable is solved

$$[M]^{-1}[F] = [M]^{-1}[B][\dot{q}] + [M]^{-1}[K][q] + [\ddot{q}_d] - \lambda[\ddot{q}]$$

$$[F] = [B][\dot{q}] + [K][q] + [M][\ddot{q}_d] - [M]\lambda[\ddot{q}]$$

$$[\widehat{U}] = [B][\dot{q}] + [K][q] + [M][\ddot{q}_d] - [M]\lambda[\ddot{q}]$$
(9)

With this, a control signal would have been found, however, considering that the system analyzed is subject to disturbances, it is necessary to add a term to the control signal in a way that compensates for this phenomenon.

$$u = [\hat{u} - k_s sgn(S)] \tag{10}$$

Where sgn, represent the sign function which varies according to

$$sgn(s) = \begin{cases} 1 & s > 0 \\ -1 & s < 0 \end{cases}$$
 (11)

In this case, the control signal would be given by,

$$[U] = [B][\dot{q}] + [K][q] + [M][\ddot{q}_d] - [M]\lambda[\dot{\tilde{q}}] - [k_s]sgn(S)$$
(12)

Now, to verify stability, a Lyapunov candidate function must be used, according to the following general expression,

$$V = \frac{1}{2}S^2 \tag{13}$$

fulfilling that \dot{V} must be defined negative, then,

$$\dot{V} = \dot{S}S < 0 \tag{14}$$

So, in this case, the derivative of the candidate Lyapunov function is,

$$\dot{V} = -[M]^{-1}[k_s]sgn(S)S$$
(15)

If, sgn(S)S = |S|, and the matrix $[k_s]$ is composed, in addition to gains, by [M] so,

$$V = -[k_s]|S|$$

$$\dot{V} = -[k_s]|[\dot{q}] - [\dot{q}_d] + \lambda[\tilde{q}]|$$
(16)

This verifies the stability of the system. However, as mentioned above, the dynamic equation of a typical system was considered. Now comparing it with the expression that generally represents the dynamics of a parallel robot (see equation (17)), it is observed that these have the same shape, also considering that the order of the system is the same, the sliding surface and its derivative, are given by expressions (3) and (5) respectively. Solving the highest order derivative of equation (17), expression (18) is obtained.

$$[H][F] = [M][\ddot{q}] + [C][\dot{q}] + [G] + [F_C]$$
(17)

$$[\ddot{q}] = [M]^{-1}[H][F] - [M]^{-1}[C][\dot{q}] - [M]^{-1}[G] + [M]^{-1}[F_C]$$
(18)

It is important to note that in the equation (17), [M] is the inertia matrix, [C] represents the vector of coriolis and centripetal forces, [G] is the vector of gravitational forces, $[F_C]$ corresponds to external forces, [H] is a matrix of unit vectors and [F] are the internal forces exerted by the actuator.

Replacing the equation (18) in (5), considering that $\dot{S} = 0$, making $[F] = [\hat{U}]$ and solving it,

$$\left[\widehat{U}\right] = [H]^{-1}[C][\dot{q}] + [H]^{-1}[G] + [H]^{-1}[F_C] + [H]^{-1}[M][\ddot{q}_d] - [H]^{-1}[M]\lambda[\dot{\tilde{q}}]$$
(19)

Now, considering the equation (10), proposed for the compensation of uncertainty, the control law for the robot would be given by,

$$[U] = [H]^{-1}[C][\dot{q}] + [H]^{-1}[G] + [H]^{-1}[F_C] + [H]^{-1}[M][\ddot{q}_d] - [H]^{-1}[M]\lambda[\dot{\tilde{q}}] - [k_s]sgn(S)$$
(20)

In terms of stability, as in the case of the Mass - Spring - Damper system, it is verified that it is stable, since the same thing happens, that is, \dot{V} is negative definite. However, when carrying out the simulation with this control law (20), an undesirable phenomenon occurs, known as chattering. Although it follows the trajectory, it is not recommended for the system to oscillate in this way. Due to the dynamics of the system and the desired trajectory, chattering is not a tolerable phenomenon, for this reason, it is necessary to reduce this effect, determining a boundary layer around the selected sliding surface, in this way, the control signal would be given by,

$$u = \hat{u} - k_s sat(\frac{S}{\Phi}) \tag{21}$$

Where, Φ represent the thickness of the plate and the term $sat(\frac{s}{\Phi})$ implies that,

$$sat(y) = \begin{cases} y & |y| \le 1\\ sgn(y) & contrary \end{cases}$$
(22)

When carrying out the simulation with this new control signal, it is found that chattering is decreased.

5. Simulations

For the controller simulations, three different types of trajectories are analyzed: step, sinusoidal and polynomial. As a first measure, a test is carried out, using the sign function, observing the behavior and the error, as shown in Figure 2.





a. Test trajectory tracking



b. Error test trajectory

Source: Own.

Now, the performance of the controller with the proposed layer to reduce chattering for the same test trajectory is shown in Figure 3.

Figure 2 Controller Performance - Function sat – Test Trajectory



For the step input, the behavior of each of the robot's actuators is analyzed, within the robot's workspace, obtaining the results of Figure 4.



Figura 3 Controller Performance - First Trajectory

a. Trajectory tracking – Actuator 1



b. Trajectory tracking – Actuator 2



For a sinusoidal input, the performance shown in Figure 5 is obtained.



Figure 4 Controller Performance – Second Trajectory

a. Trajectory tracking – Actuator 1



b. Trajectory tracking – Actuator 2



c. Trajectory tracking – Actuator 3 Source: Own.

Finally, the performance of the controller for a polynomial desired trajectory with the same perturbation used in calculated torque control [9],



Figure 5 Controller Performance – Third Trajectory



c. Trajectory tracking - Actuator 3 Source: Own.

Once the simulations have been carried out and the error calculated, the results are analyzed and compared with the computed torque control strategy.

6. Analysis of Results

To establish the comparison, the errors obtained in each trajectory for each of the robot's legs are analyzed.

Starting with the first trajectory, the errors are shown in Figure 7, in which it is observed that the performance of the controllers is very similar, the only notable difference is the response time of the first actuator, since with control by modes sliding reaches the desired value in a shorter time.

a. Trajectory error – Actuator 1 – Computed torque



c. Trajectory error - Actuator 2 - Computed torque



e. Trajectory error – Actuator 3 – Computed torque



b. Trajectory error - Actuator 1 - SMC



d. Trajectory error – Actuator 2 – SMC





Figure 6 Comparison of errors – First trajectory

Source: Own.

As for the second trajectory, the results are shown in Figure 8, which shows that the control by sliding modes presents better performance for all actuators because it has a smaller error.

Figure 7 Comparison of errors – First trajectory



a. Trajectory error - Actuator 1 - Computed torque



c. Trajectory error – Actuator 2 – Computed torque



e. Trajectory error – Actuator 3 – Computed torque Source: Own.



b. Trajectory error - Actuator 1 - SMC



d. Trajectory error - Actuator 2 - SMC



f. Trajectory error – Actuator 3 – SMC

Finally, the errors for the polynomial trajectory are compared as seen in Figure 9, where is evident that the control by sliding modes presents better performance, particularly for the first two actuators due these presents a smaller error, compared to the torque control. computed.

Figure 8 Comparison of errors – Third trajectory



a. Trajectory error – Actuator 1 – Computed torque



c. Trajectory error – Actuator 2 – Computed torque



e. Trajectory error – Actuator 3 – Computed torque



b. Trajectory error - Actuator 1 - SMC



d. Trajectory error - Actuator 2 - SMC



f. Trajectory error – Actuator 3 – SMC



7. Conclusions

Parallel robots have non-linear and complex dynamics, which implies that robust control strategies are used to better leverage their advantages. In this article, a control law was

designed, considering the theory of sliding modes, analyzing its performance applied to a 3SPS – 1U parallel robot through simulation carried out in Matlab with three different trajectories, analyzing its error by comparing it with the obtained for a computed torque control obtained from another publication.

The first trajectory was linear, corresponding to the step. However, the magnitude of the signal was changed because if we worked with the unit step it was not possible to observe the behavior of the controller in detail; It should be noted that for such a low value, the performance of the controller is very good and due to the robustness of the platform and the application, which is the simulation of movement, it was about observing what happened when the rod protruded from the cylinder for a considerable length, in accordance with its maximum displacement.

The same was done with the sinusoidal and polynomial input.

According to the results found, it was evident that, for linear input, computed torque control has good performance in a parallel robot; however, for the other inputs considered, the control by sliding modes was the one that presented the best performance showing a smaller error, even in the presence of disturbances.

In this way, it is demonstrated that for a 3DOF parallel robot oriented to the simulation of movement subject to disturbances, the control by sliding modes is the one that has the best performance compared to a control by computed torque.

Acknowledgements

We extend our heartfelt gratitude to the University Industrial of Santander and the University Santo Tomas for their invaluable support and facilitation of this research project. Their commitment to fostering knowledge and innovation in the field of robotics has been instrumental in the successful completion of this study. We are deeply appreciative of the resources, guidance, and encouragement provided by these esteemed institutions, which have significantly contributed to advancing our understanding and expertise in robotics and of course

to the University Distrital Francisco Jose de Caldas that opens spaces for the dissemination of

results.

References

- [1] M. Garcia Sanz and M. Motilva Casado, "Herramientas para el estudio de robots de cinemática paralela: Simulador y prototipo experimental," *Revista Iberoamericana de Automática e Informática Industrial, RIAI,* vol. 2, no. 2, pp. 73-81, 2005. <u>https://polipapers.upv.es/index.php/RIAI/article/view/8064</u>
- [2] A. I. Aureles Cabrera, Robot paralelo tipo STEWART para la rehabilitación de tobillo, Hidalgo, Mexico: Universidad Politécnica de Tulancingo, 2019. <u>http://www.upt.edu.mx/Contenido/Investigacion/Contenido/TESIS/MAC/2019/MAC_T_2</u> 019_01_AAC.pdf
- [3] Instituto de Investigación de Seguridad en la Conducción IOWA, «Simulador NADS 1,» Univesidad de Iowa, 2023. [En línea]. Available: <u>https://dsri.uiowa.edu/nads-1</u>. [Último acceso: 02 2023].
- [4] SIMAERO, "AIRBUS A340 FFS," SIMAERO, 2023. [Online]. Available: https://www.sim.aero/a340/. [Último acceso 02 2023].
- [5] O. Altuzarra, Y. San Martín, E. Amezua and A. Hernández, "Motion pattern analysis of parallel kinematic machines: A case study," *Robotics and Computer-Integrated Manufacturing*, vol. 25, no. 2, pp. 432-440, 2009. <u>https://doi.org/10.1016/j.rcim.2008.01.007</u>
- [6] J. Fernandes and A. Selvakumar, "Kinematic and Dynamic Analysis of 3PUU Parallel Manipulator for Medical Applications," *Proceedia Computer Science*, vol. 133, no. 1, pp. 604-611, 2018. <u>https://doi.org/10.1016/j.procs.2018.07.091</u>
- [7] I. Ben Hamida, M. Amine Laribi, A. Mlika, L. Romdhane, S. Zeghloul and G. Carbone, "Multi-Objective optimal design of a cable driven parallel robot for rehabilitation tasks," *Mechanism and Machine Theory*, vol. 156, no. 1, pp. 104-141, 2021. <u>https://doi.org/10.1016/j.mechmachtheory.2020.104141</u>
- [8] K. Duarte Barón and C. Borrás Pinilla, «Generalidades de robots paralelos,» Revista visión electrónica, algo más que un estado sólido, vol. 10, nº 1, pp. 1-11, 2016. <u>https://doi.org/10.14483/22484728.11711</u>

- [9] K. Duarte Barón, C. Borrás Pinilla and J. J. Gil Pelaez, «Dynamic analysis and simulation of computed torque control of a parallel robot 3SPS - 1U,» de IEEE 4th Colombian Conference on Automatic Control (CCAC), Medellín, Colombia, 2019. <u>https://doi.org/10.1109/CCAC.2019.8921238</u>
- [10] C. Gosselin and J. Angeles, "Singularity analysis of closed-loop kinematic chains," IEEE Transactions on Robotics and Automation, vol. 6, no. 3, pp. 281-290, 1990. <u>https://doi.org/10.1109/70.56660</u>
- [11] J. Kardos, "Robust Computed Torque Method of Robot Tracking Control," in 22nd International Conference on Process Control (PC19), Strbske Pleso, Slovakia, 2019. <u>https://doi.org/10.1109/PC.2019.8815088</u>
- [12] C. Jun and W. Lin, "Track Tracking of Double Joint Robot Based on Sliding Mode Control," in IEEE 3rd International Conference on Information Systems and Computer Aided Education (ICISCAE), Dalian, China, 2020. <u>https://doi.org/10.1109/ICISCAE51034.2020.9236895</u>
- [13] W. X. Xu, G. Z. Cao, Y. P. Zhang, J. C. Chen, D. P. Tan and Z. Q. Ling, "Adaptive backstepping sliding mode control of lower limb exoskele-ton robot based on combined double power reaching law," in 2th International Conference on CYBER Technology in Automation, Control, and Intelligent Systems (CYBER), Baishan, China, 2022. https://doi.org/10.1109/CYBER55403.2022.9907279
- [14] X. Chen, H. Chen, Y. Huang and Q. Huang, "Adaptability Control Towards Complex Ground Based on Fuzzy Logic for Humanoid Robots," *IEEE Transactions on Fuzzy Systems*, vol. 30, no. 6, pp. 1574-1584, 2022. https://doi.org/10.1109/TFUZZ.2022.3167458
- [15] D. Li, J. Pan, J. Liu, M. Wang and J. Yu, "Model Predictive Control Based Path Following of an Amphibious Robot," in *0th Chinese Control Conference (CCC)*, 2021. <u>https://doi.org/10.23919/CCC52363.2021.9549348</u>
- [16] Y. Zhang, L. Sol and Y. Zhang, "Research on Algorithm of Humanoid Robot Arm Control System Based on Fuzzy PID Control," in *International Conference on Artificial Intelligence and Autonomous Robot Systems (AIARS)*, Bristol, United Kingdom, 2022. <u>https://doi.org/10.1109/AIARS57204.2022.00082</u>
- [17] K. Duarte Barón and C. Borrás Pinilla, Analisis, diseño y simulacion de un control robusto para un robot paralelo de 3 grados de libertad, Bucaramanga, Colombia, Universidad Industrial de Santander, 2019. <u>https://noesis.uis.edu.co/items/c91bc6a4-e228-44f8-8ab4-33000e9e8688</u>
- [18] J. J. Slotine and W. Li, Applied nonlinear control, New Jersey: Prentice Hall, 1991.

- [19] S. Iqbal and A. I. Bhatti, "Robust sliding-mode controller design for a stewart platform," in Proceedings of International Bhurban Conference on Applied Sciences, Islamabad, Pakistan, 2007. <u>https://doi.org/10.1109/IBCAST.2007.4379924</u>
- [20] C. Zhang and L. Zhang, "Kinematics analysis and workspace investigation of a novel 2-DOF parallel manipulator applied in vehicle driving simulator," *Robotics and Computer-Integrated Manufacturing*, vol. 29, no. 2, pp. 113-120, 2013. <u>https://doi.org/10.1016/j.rcim.2012.11.005</u>