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Index profile model for the synthesis of few-mode multimode optical fibers

Modelo de perfil de índice para la síntesis de fibras ópticas de pocos modos

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Abstract

This paper presents the development of a model that allows the calculation and design of multimode optical fibers of few modes, from the mechanical parameters that describe the geometry of the parabolic index profile and the electromagnetic behavior in this guided transmission medium. For this purpose, a system of equations has been developed based on the performance evaluation metrics of an optical fiber and an independent variable that is adjusted through an optimization algorithm to evaluate the best values of bandwidth, dispersion, and propagated modes in multimode optical fiber propagation windows.

Keywords: Index, Profile, Bandwidth, Mode, Multimode, Parabolic.

Resumen

Este documento presenta el desarrollo de un modelo que permite el cálculo y diseño de fibras ópticas multimodo de pocos modos a partir de los parámetros mecánicos que describen la

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geometría del perfil de índice parabólico y el comportamiento electromagnético de este medio de transmisión guiado. para ello se han conformado sistemas de ecuaciones basados en las métricas de evaluación de desempeño de una fibra óptica y una variable independiente que es ajustada por medio de un algoritmo de optimización para evaluar los mejores valores de ancho de banda, dispersión y modos propagados en las ventanas de propagación de fibra óptica multimodo.

Palabras clave: Índice, Perfil, Ancho de banda, Modo, Multimodo, Parabólico

1. Introduction

Over the past few years, a new fiber material has been developed with the potential of making better use of communication channels compared to the mono-mode fiber used in most network links. The new material is a multimode fiber, yet it contains fewer modes than the conventional fiber.

Depending on its primary feature, which defines the name of the same fiber, it offers the possibility to adapt to the phenomena seen in standard multimode fibers.

According to [1], [2], [3], and other articles, the number of modes that can be propagated by this type of fiber depends on the radius of its core, as the wavelength with which a beam of light is propagated may or may not enter the fiber given this parameter. This occurs when the propagation of the fundamental mode LP01 [4] takes place in any optic fiber.

As seen in [5], the intrinsic parameters used to design fibers with fewer modes include the radius, the index profile, and the fiber length [6][7]. In essence, the radius of the core enables to modify the number of propagated modes [3][8][9], the index profile also has an effect over the number of modes and the index value of the fiber; as such, the step-based nature of the geometry can account for a higher number of modes compared to its gradual counterpart [1] [10]. A higher number of modes is also related to an increase in their modal dispersion [11][12][13] and the distance travelled by the modes to be transported [1].

The measurement of modes and signal quality in optic fibers takes into consideration the attenuation, dispersion, and latency of modes, among other variables. Attenuation can significantly affect the length with which a link can be established, and it increases proportionally with the distance [14], meaning that the attenuation factor is measured in $\frac{dB}{km}$[3][8]. Dispersion is one of the most important factors regarding the transmission of an information signal. This factor is categorized into two types: modal and chromatic. Modal dispersion is mostly present in optic fibers that handle more than one mode, and chromatic dispersion affects all fibers [3][8]. The latency between modes is measured based on the propagation time of the modes throughout a section of fiber [5].

2. Materials and methods

2.1. Propagation modes

When a fiber can tolerate numerous modes, it is called a multimode fiber. The number of modes is given by the normalized frequency [15] [16].

$$V = \frac{2\pi d}{\lambda}\sqrt{{n_1}^2 - {n_2}^2} \ (1)$$

where d is the diameter of the fiber and λ is the wavelength of the optic fiber.

Based on equation 1, the number N of modes propagated by a step-index fiber can be determined as:

$$M = \frac{V^2}{2} \qquad (2)$$

The number of modes in a step-index optic fiber can be determined through equation 3, The dependance on the variable λ is noticeable, since it defines the geometry of the profile:

$$M = \frac{\alpha}{\alpha + 2} a^2 {k_0}^2 \Delta \qquad (3)$$

2.1.1. Index difference

The parameter Δ is defined as the difference between the indexes of the core and the coating, where the typical values of the multimode fibers vary between 1% and 3%, so the value of Δ is significantly lower than 1 [15][17].

$$\Delta = \frac{{n_1}^2 - {n_2}^2}{2{n_1}^2} \qquad (4)$$

2.1.2. Wave number

It is the magnitude that represents the number of cycles completed by the wave per distance unit. It is calculated by dividing 2π by the wavelength.

$$k = \frac{2\pi}{\lambda} \left[\frac{rad}{m}\right] \qquad (5)$$

In the field of physics, the wave number is a parameter of the stationary wave equation (equation 6) that multiplies the position within the sinus argument. It is a characteristic value of this type of wave.

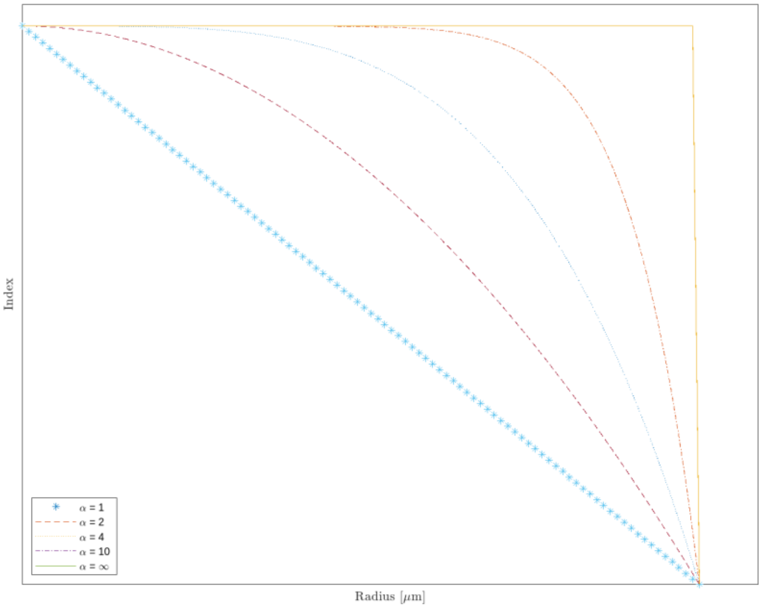
$$y = 2A\ \sin(k_0 x)\cos(wt) \qquad (6)$$

2.1.3. Alpha equation

The multimode fibers of gradual index can be defined through equation 7, which can be used to describe the geometric behavior of the core index in terms of the variation of the radius through the core (parabolic profile) [18], based on a constant value of the fiber coating index. The fiber profile has a cone shape when the parameter $\alpha = 1$ and, when it is parabolic, α is close to 2. For larger or infinity-tending values of α, the profile describes the behavior of a step-index fiber (Figure 1).

$$n(r) = \begin{cases} n_1\sqrt{1 - 2\Delta\left(\frac{r}{a}\right)^{\alpha}} & for\ r < a \\ n_1\sqrt{1 - 2\Delta} & for\ r > a \end{cases} \qquad (7)$$

Figure 1. Index profile for variations of α



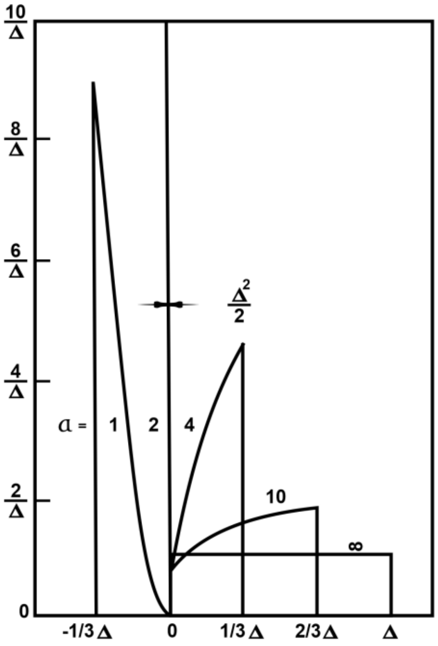
Source: own.

2.1.4. Optimal alpha

It is the condition in which the inferior and superior modes reach time $t = 0$. According to the impulse response (Figure 2) for different profiles with gradual index (other than α) [19].

$$\alpha = 2(1 - \Delta) \qquad (8)$$

Figure 2. Relative delay in response to impulse in multimode fibers.



Source: [19]

2.1.5. Propagation constant for guided modes

A mode exists (propagates) when its propagation constant remains between the value of the core index and the coating index (equation 9). On the other hand, the normalized propagation constant b [20] varies between 0 and 1, in terms of the normalized frequency (equation 1) [16].

Hence:

$$n_2 k \; < \; \beta \; < \; n_1 k \qquad (9)$$

$$b = 1 - \frac{u^2}{V^2} \qquad (10)$$

where:

$$u = R\,\sqrt{k_0 {n_1}^2 - \beta^2} \qquad (11)$$

and:

$$k_0 = \beta max = \frac{2\pi n_1}{\lambda_c}\left[\frac{Rad}{m}\right] \qquad (12)$$

2.2. Modal dispersion

Modal dispersion (or pulse expansion) is due to the difference in the propagation times of light rays that travel different trajectories in a fiber [18].

$$D = \frac{\Delta T}{L} = \frac{n_1 \Delta}{8c}\;[\frac{s}{m}] \qquad (13)$$

Source: [19][20]

3. Development of the model

The design of a multimode optic fiber requires determining its physical (mechanical) parameters, such as the core radius, the refractive indexes of the core and the coating, and the propagation parameters, such as the frequency of operation and the number of modes that can be propagated.

The model proposed in this article involves two equation systems (14 and 15) that depend on the core refractive index, the parameter α, the core radius, the propagated modes, the wavelength of the fiber, and the constant of the speed of light.

3.1. System of equations in terms of the number of modes.

Based on the differential equations of indexes k_0, substitutions are carried out in the equations of optimal alpha and the dispersion (D) to generate the dependance in terms of the number of modes (M) as shown in the equation system (14).

$$\alpha, D(M)\begin{cases} \alpha = \dfrac{2M\lambda^2}{2(a\pi)^2({n_1}^2 - {n_2}^2) - M\lambda^2} \\ D = \dfrac{n_1}{8c}\left(1 - \dfrac{M\lambda^2}{2(a\pi)^2({n_1}^2 - {n_2}^2) - M\lambda^2}\right) \end{cases} \tag{14}$$

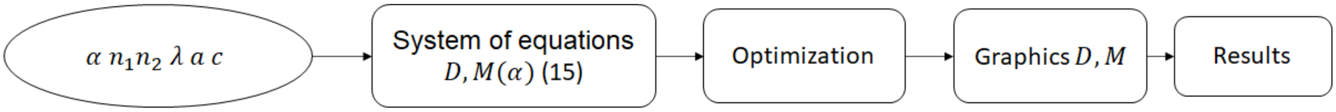
3.2. System of equations in terms of the parameter α

Based on the differential equations of indexes k_0 and optimal α, substitutions are carried out in the equations of the propagated modes and dispersion to generate the dependance in terms of the number of modes.

$$D,M(\alpha)\begin{cases} D = \dfrac{n_1}{32c}(\alpha - 2)^2 \\ M = \dfrac{\alpha}{\alpha + 2}\left(\dfrac{2a^2\pi^2}{\lambda^2}\right)\left({n_1}^2 - {n_2}^2\right) \end{cases} \tag{15}$$

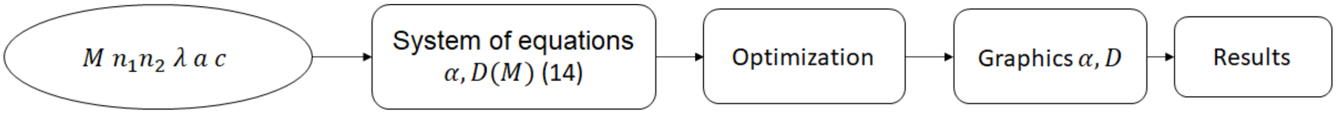
Figure 3 shows a diagram flow of the processes for adjusting values in the equation systems for each case to be implemented in Matlab.

Figure 3. Design based on the variation of $\boldsymbol{\alpha}$



Source: own.

Figure 4. Design based on the variation of $\boldsymbol{M}$



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4. Results and analysis

The results obtained in the context of optic fibers of few modes using the proposed model are presented below in Table 1 and Table 2, respectively.

Table 1. Results of the model optimization in terms of α.

n_1	n_2	Radius (μm)	$\lambda[nm]$	α	Modes	Dispersion [s/m]	BW [(Gb/s)x km]
1,48	1,478	20	850	0,2	5	4,99E-10	0,002
				0,6	15	2,83E-10	0,0035
				1,1913	24	1,01E-10	0,00991
				1,6600	29	1,77E-11	0,0564
				1,9790	32	6,22E-14	16,07
1,48	1,478	20	1330	0,2090	2	4,95E-10	0,00202
				0,6533	6	2,80E-10	0,003576
				1,1355	9	1,15E-10	0,008679
				1,6520	11	1,86E-11	0,0536
				1,9790	13	6,24E-14	16,03
1,48	1,47	20	850	0,2080	30	4,95E-10	0,00202
				0,6114	75	2,97E-10	0,00336
				1,1470	117	1,21E-10	0,00892
				1,6190	144	2,24E-11	0,0447
				1,9800	160	6,17E-14	16,21
1,48	1,47	20	1330	0,2023	12	4,98E-10	0,002007
				0,6313	31	2,89E-10	0,003462
				1,1571	48	1,10E-10	0,009129
				1,5420	57	3,23E-11	0,0309
				1,9782	65	7,36E-14	13,59
1,48	1,478	30	850	0,2147	14	4,91E-10	0,002035
				0,6270	34	2,91E-10	0,00344
				1,1111	51	1,22E-10	0,0082
				1,6050	64	2,40E-11	0,04171
				1,9798	72	6,29E-14	15,9
1,48	1,478	30	1330	0,2012	5	4,99E-10	0,002004
				0,6250	14	2,91E-10	0,00343
				1,1104	21	1,22E-10	0,008196
				1,6560	26	1,82E-11	0,05489
				1,9780	29	6,92E-14	14,44

Source: own.

Table 2. Results of the model optimization in terms of the propagated modes.

n_1	n_2	Radius (μm)	$\lambda[nm]$	α	Modes	Dispersion [s/m]	BW [(Gb/s)x km]
1,48	1,478	20	850	0,2121	6	4,93E-10	0,00202
				0,60205	14	3,01E-10	0,00331
				1,0626	22	1,35E-10	0,00738
				1,6190	28	2,23E-11	0,0448
				1,9883	32	2,12E-14	47,1
1,48	1,478	20	1330	0,5700	5	3,15E-10	0,00317
				0,8347	7	2,09E-10	0,00477
				1,1100	9	1,20E-10	0,00832
				1,4690	11	4,33E-11	0,02308
				1,9717	13	1,23E-13	8,11
1,48	1,47	20	850	0,1770	26	5,12E-10	0,001951
				0,4934	63	3,50E-10	0,002858
				0,9450	103	1,72E-10	0,00582
				1,5794	142	2,73E-11	0,03665
				1,9880	160	2,18E-14	45,89
1,48	1,47	20	1330	0,2634	15	4,70E-10	0,00215
				0,5607	28	3,19E-10	0,003131
				0,8524	39	2,03E-10	0,004925
				1,5140	56	3,64E-11	0,0275
				1,9860	65	2,71E-14	36,8
1,48	1,478	30	850	0,2965	18	4,47E-10	0,00223
				0,5130	29	3,41E-10	0,00293
				0,9930	48	1,56E-10	0,06401
				1,4380	60	4,86E-11	0,0205
				1,9790	72	6,40E-14	15,6
1,48	1,478	30	1330	0,2008	5	4,99E-10	0,002
				0,6325	14	2,88E-10	0,00346
				0,9215	18	1,79E-10	0,00557
				1,6560	26	1,82E-11	0,055
				1,9750	29	9,17E-14	10,9

Source: own.

The analysis of the optical fiber with a radius of 20 μm and low index differences ($n_1$and n_2 close) leads to the conclusion that an optic fiber with an operation length of 1330 nm, a wavelength of 850 nm, and a similar value of α can handle at least twice as many modes compared to a multi-mode fiber for 1330 nm.

When designing a multi-mode fiber for an operation length of 850 nm, the bandwidth (BW) is higher than for a multi-mode fiber with an operational length of 1330 nm when α is similar for both fiber designs. The comparison of the results obtained with higher index differences and the initial designs shows an increase in the number of modes supported for the wavelength values of 850 and 1330 nm.

The design of multi-mode fibers for a wavelength of 850 nm and a higher index difference assumes that the dispersion and bandwidth are similar to those used in the design of fibers with lower index differences. Furthermore, in multi-mode fibers with a wavelength of 1330 nm, the dispersion is reduced and the bandwidth increases compared to multi-mode fibers of 1330 nm and low index differences.

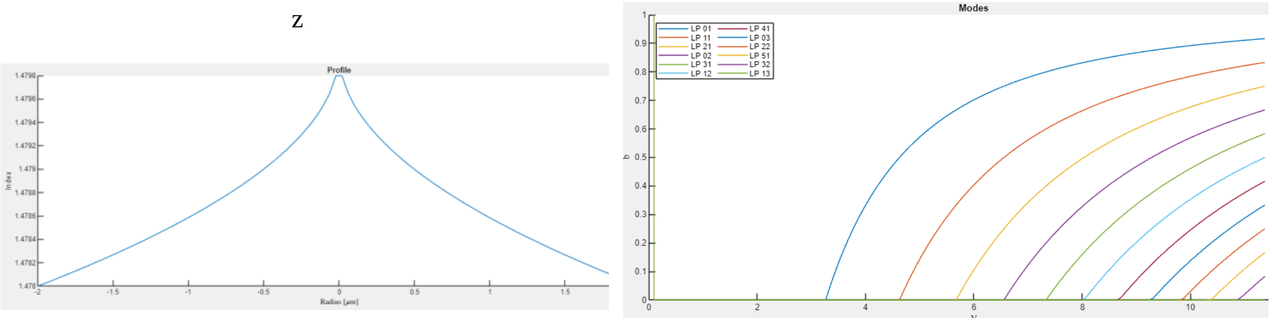
When changing the fiber radius from 20 to 30 μm with the same value of α, the number of modes of a fiber with a wavelength of 850 nm is similar to those in a multi-mode fiber with a wavelength of 1330 nm, a 20 μm radius, and distant indexes $n_1$and n_2. However, the BW is reduced, and the dispersion increases for a radius of 30 μm.

A fiber with a wavelength of 1330 nm and a 30 μm radius exhibits a similar behavior in the number of modes for the same values of α in a fiber designed for a wavelength of 850 nm and a radius of 20 μm. Additionally, the BW is reduced, and the dispersion increases for a fiber with a 30 μm radius given that the number of modes increases.

The dispersion value is higher for both operational wavelengths when α < 1.9 since there is more confinement due to the geometry of the index profiles (Figures 5a, 6a, and 7a), which minimizes the number of modes and increases the separation between them (Figures 5b, 6b, and 7b).

In this section, three designs are presented where α varies for a radius of 20μm, $\lambda = 850nm$, n_1 =1.48 and n_2 =1.478. These parameters lead to a gradual index fiber with a profile that is neither parabolic nor rectangular, given that α < 1. This set of parameters results in a maximum of 12 propagated modes (Figure 5b) where the last sample of the normalized frequency sets the propagation of the twelfth mode (LP 13).

Figure 5. Design of an optic fiber profile for a 20μm radius, $\alpha = 0.5$, n_1 =1.48 and n_2 =1.478.



$(\boldsymbol{a})\ \boldsymbol{Profile\ with\ \alpha = 0.5}$

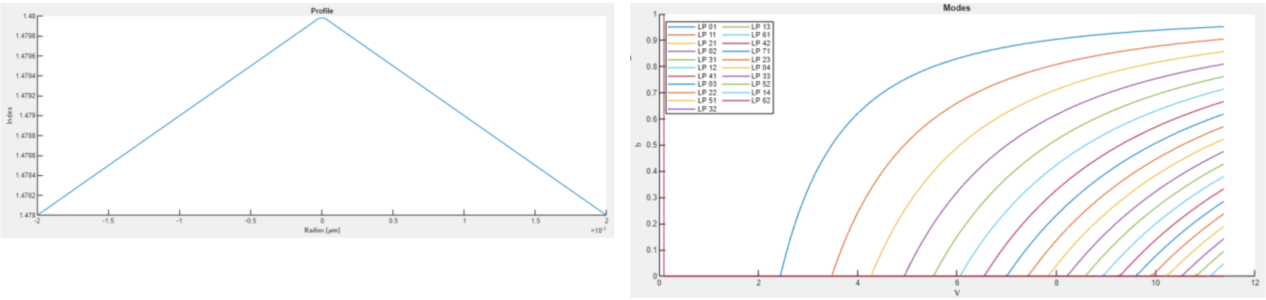
$(\boldsymbol{b})\ \boldsymbol{Propagated\ modes\ with\ \alpha = 0.5}$

$\boldsymbol{y\ \lambda = 850nm}$

Source: own.

Setting the parameter α to 1, results in a gradual rectangular profile. This set of parameters delivers a maximum of 21 propagated modes, as seen in Figure 6b, where the last sample of normalized frequency sets the propagation of the 23rd mode (LP 62).

Figure 6. Design of an optic fiber profile for a 20μm radius, $\alpha = 1$, n_1 =1.48 and n_1 =1.478.



$(\boldsymbol{a})\ \boldsymbol{perfil\ con\ \alpha = 1}$

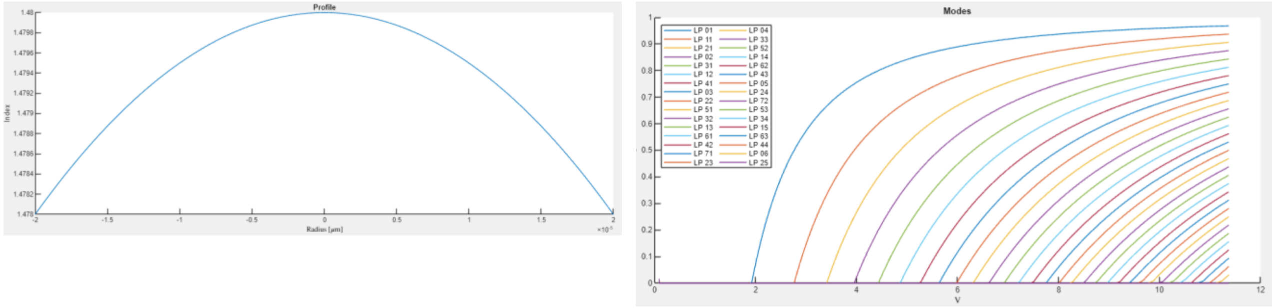
$(\boldsymbol{b})\ \boldsymbol{Modos\ propagados\ con\ \alpha = 1}$

$\boldsymbol{y\ \lambda = 850nm}$

Source: own..

Lastly, when α = 2, a gradual parabolic profile is obtained. This set of parameters delivers a maximum of 32 propagated modes as seen in Figure 7b where the last sample of normalized frequency sets the propagation of the 32nd mode (LP 25).

Figure 7. Design of an optic fiber profile for a 20μm radius, $\alpha = 2$, n_1 =1.48 and n_1 =1.478.



$(\boldsymbol{a})$ $\boldsymbol{Profile\ with\ \alpha = 2}$

$(\boldsymbol{b})$ $\boldsymbol{Propagated\ modes\ with\ \alpha = 2}$

$\boldsymbol{y\ \lambda = 850nm}$

Source: own.

5. Conclusion

It was possible to identify that, for a 0.6% difference in refractive indexes, the number of modes that can be propagated by a gradual index fiber is lower compared to fibers with index differences of 0.1%, as for α < 1.9. This sets the geometry of the parabolic index of an optic fiber, and the gradual index difference below 1% translates into bandwidths below 1 $(Gb/s)km$. When the radius length of the core (refractive index n_1) ranges between 20 um and 30 um, the number of modes admitted by the optic fiber increases as the length increases. On the other hand, when $\lambda = 850\ nm$, the number of modes supported by the optic fiber is higher than for optic fibers with $\lambda = 1330\ nm$.

Two equation systems were obtained using the parameters defined in the theory of multi-mode fibers with gradual index. Said systems were proposed based on the dispersion equations of multi-mode fibers, optimal α, and mode calculations, expressing the parameter α and the number of modes as independent variables.

These systems determine the behavior based on length unit and fiber modes for a given set of design parameters, to be then introduced in the modeling of the index profile for a gradual index fiber.

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